READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

Silly 10 Point Bonus: The magnitude of error in using Simpson's Rule to approximate the definite integral of f(x) on an interval [a,b] with n subintervals may be estimated using

$$\left| \int_a^b f(x)\,dx - S_n \right| = |ES_n| \le \frac{K_4(b-a)^5}{180n^4},$$

provided the fourth derivative of f, $f^{(4)}(x)$, is continuous on [a,b], and $|f^{(4)}(x)| \le K_4$ for each x in [a,b]. If Simpson's rule is used to approximate the definite integral giving the exact value of ln(2), reveal how to determine how big must n be to ensure you have 2 decimal place accuracy. [Say where your work is for there isn't room here. Remember n must be even.]

The integral that we must approximate is

$$\ln(2) = \int_1^2 \frac{1}{x}\,dx.$$

Here, the integrand is $f(x) = x^{-1}$, and thus,

$$f'(x) = -x^{-2},\ f''(x) = 2x^{-3},\ f^{(3)}(x) = -6x^{-4},\ \textit{and}\ f^{(4)}(x) = 24x^{-5}.$$

To use the error estimate really effectively, it helps to obtain the maximum of

$$|f^{(4)}(x)| = |24x^{-5}| = \frac{24}{x^5}$$

with $x \in [1,2]$. This, of course, is easy since $|f^{(4)}(x)|$ above is decreasing on the interval [1,2]. Consequently, for $x \in [1,2]$, we have $|f^{(4)}(x)| \le |f^{(4)}(1)| = 24$.

This means that to obtain the desired accuracy, by using the error bound provided up the page, it suffices to find an even positive integer n so that

$$(*) \qquad \frac{24}{180n^4} < \frac{1}{2}10^{-2}$$

is true.

It is easy to see that inequality (*) above is equivalent to

$$\left(\frac{80}{3}\right)^{\frac{1}{4}} < n.$$

Consequently, since $80/3 < 81 = 3^4$ and 4th roots preserve order, the smallest even number that makes (*) true is $n = 4$.//

Are you suffering from epsilon ennui yet??