NAME: **OgreOgre [Bonus.]** **TEST3/MAC2312**

READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page. Eschew obfuscation.*

Silly 10 Point Bonus: Obtain an infinite series that gives the exact value of $\tan^{-1}(2)$. Say where your work is, for it won't fit here!!

Since the Maclaurin series for $\tan^{-1}$ only converges for $x \in [-1,1]$, what's a poor ogre to do?? Ah, yes, there was that bonus problem on Test 1 this term. We obtained

(1) $$\int_0^x \frac{1}{1+t^2}\,dt + \int_0^{1/x} \frac{1}{1+t^2}\,dt = 2\int_0^1 \frac{1}{1+t^2}\,dt$$

for each $x > 0$. This, of course, implies that

$$\tan^{-1}(x) + \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}$$

for each $x > 0$. Thus,

$$\begin{aligned}\tan^{-1}(2) &= \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}\right)\\ &= \frac{\pi}{2} - \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}\left(\frac{1}{2}\right)^{2k+1}\\ &= \frac{\pi}{2} + \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{2k+1}\left(\frac{1}{2}\right)^{2k+1}.\end{aligned}$$