READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page. Eschew obfuscation.*

1. (20 pts.) Obtain the exact numerical value of each of the following if possible. If a limit doesn't exist or an improper integral or an infinite series fails to converge, say so.

(a) $\int_2^{\infty} \frac{10}{x(x+1)}\, dx =$

(b) $\sum_{k=2}^{\infty} \frac{10}{k(k+1)} =$

(c) $\int_1^{2} \frac{1}{(2-x)}\, dx =$

(d) $\sum_{k=0}^{\infty} 10\left(-\frac{3}{4}\right)^k =$

(e) $\lim_{n\to\infty} \left(1 + \frac{\ln(3)}{n}\right)^n =$

2. (20 pts.) With proof, determine whether each of the following infinite series converge. If a series converges, do not attempt to obtain its sum.

(a) $$\sum_{k=1}^{\infty} \frac{10}{2 + 3k^2}$$

(b) $$\sum_{k=1}^{\infty} \frac{10}{2 + \frac{3}{k^2}}$$

(c) $$\sum_{k=1}^{\infty} \frac{10^k}{3k!}$$

(d) $$\sum_{k=1}^{\infty} \left(\frac{1}{2 + \frac{3}{k^2}} \right)^k$$

(e) $$\sum_{k=2}^{\infty} \frac{10}{k \ln^2(k)}$$

3. (12 pts.) Each of the following power series functions is the Maclaurin series of some well-known function. In each case, (i) identify the function, and (ii) provide the interval in which the series actually converges to the function.

(a) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} =$

(b) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1} =$

(c) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} =$

(d) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{k+1}}{k+1} =$

(e) $\sum_{k=0}^{\infty} \frac{x^k}{k!} =$

(f) $\sum_{k=0}^{\infty} x^k =$

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4. (8 pts.) Suppose that

$$f(x) = \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2 7^k}(x-1)^k$$

Find the radius of convergence and the interval of convergence of the power series function f.

5. (5 pts.) Obtain the second Taylor polynomial $p_2(x)$ of the function

$$f(x) = x^{1/3}$$

at $x_0 = 8$.

6. (5 pts.) **Using sigma notation and an appropriate Maclaurin series**, by doing term-by-term integration, obtain an infinite series that is equal to the numerical value of the following definite integral.

$$\int_0^1 \sin(x^2)\, dx =$$

7. (5 pts.) Suppose

$$f(x) = \sum_{k=1}^{\infty} \frac{2\pi(x-4)^k}{k20^k}$$

for every x ε (-16,24). By differentiating f term-by-term, obtain a power series function that is the same as f′(x). **Use sigma notation.**

$$f'(x) =$$

8. (5 pts.) Express .112112... (repeating) as the ratio of two positive integers. [**The ratio does not have to be in lowest terms.**]

.112112... =

9. (5 pts.)

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^{3/4}}$$

Prove the infinite series above is conditionally convergent.

10. (5 pts.) It turns out that

$$\int_0^1 e^{-x^2}\,dx = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)k!}.$$

To approximate the numerical value of the integral above to 2 decimal places by hand, what finite sum

$$s_n = \sum_{k=0}^{n} \frac{(-1)^k}{(2k+1)k!}.$$

should you use? Proof??

11. (5 pts.) (a) By substitution into an appropriate Maclaurin series, obtain the Maclaurin series for the function

$$f(x) = \tan^{-1}(2x^2)$$

(b) What is the domain of the function f?
(c) What is the interval of convergence for the Maclaurin series of f??

12. (5 pts.) Show how to find an interval that is symmetric about the origin where $\cos(x)$ can be approximated by $p(x) = 1 - x^2/2$ with two decimal place accuracy.

Silly 10 Point Bonus: Obtain an infinite series that gives the exact value of $\tan^{-1}(2)$. Say where your work is, for it won't fit here!!