READ ME FIRST: Show me all the magic very neatly on the page, for I do not read minds. Use correct notation when presenting your computations and arguments. Use complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer consists of all the magic transformations, do not "box" your final results. Eschew obfuscation. Each problem is worth 10 points.

---

1. Using a complete sentence and appropriate notation, state precisely the First Part of the Fundamental Theorem of Calculus. [This is sometimes called the *Evaluation Theorem*.]

---

2. Evaluate the following using the 1st Part of the Fundamental Theorem of Calculus.

$$\int_0^{1/2} \frac{1}{(1-x^2)^{1/2}}\, dx =$$

---

3. Using complete sentences and appropriate notation, state the Second Part of the Fundamental Theorem of Calculus.

---

4. (a) Obtain the solution to the following initial value problem in terms of a definite integral taken with respect to the variable ***t***, so the differential denoting the variable of integration is ***dt***.

$$\frac{dy}{dx} = \sec^2(x)e^{\tan(x)} \quad \text{with } y(\pi/4) = 5$$

(b) Then by evaluating the *dt* integral, completely identify the solution, $y(x)$.

$y(x) =$

5. (a) Give the definition of the function ln(x) in terms of a definite integral and give its domain and range. Label correctly. (**Hint**: Complete the sentence,"ln(x) = ...")

(b) Given ln(a) = -20 and ln(c) = 5, evaluate the following integral.

$$\int_a^{c^5} \frac{1}{\lambda}\, d\lambda =$$

6. If the given integral is expressed as an equivalent integral in terms of the variable u using the substitution $u = x^2 - 1$ so that the equation below is true, what are the numerical values of the new limits of integration α and β, and what is the new integrand, f(u) ??? Obtain these but do not attempt to evaluate the du integral.

$$\int_0^2 2x\,|x^2-1|^3\, dx = \int_\alpha^\beta f(u)\, du$$

du = $f(u)$ =

α = β =

7. A particle moves with a velocity of $v(t) = t^2 - 1$ along an s-axis. Find the displacement and total distance traveled over the time interval [0,2].

Displacement =

Total_Distance =

8. Find each of the following derivatives.

(a) $\dfrac{d}{dx}\left[\displaystyle\int_1^x \frac{\sin(t)}{t}\, dt \right] =$

(b) $\dfrac{d}{dx}\left[x^3 + \displaystyle\int_1^{\tan(x)} \tan^{-1}(t)\, dt \right] =$

9. Evaluate the following limits:

(*a*) $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x} =$

(*b*) $\lim_{x \to \infty} \left(1 + \frac{1}{5x}\right)^{\ln(32)x} =$

10. Let the function g be defined by the equation

$$g(x) = \int_1^x (3t^2 + 1)^{1/2}\, dt$$

for x ε (-∞ ,∞). Then

(a) g(1) =

(b) $g'(1) =$

(c) $g''(1) =$

(d) Determine the open intervals where g is increasing or decreasing. Be specific.

(d) Determine the open intervals where g is concave up or concave down. Be specific.

11. Find the exact arc length of the parametric curve without eliminating the parameter when $x = \cos(2t)$ and $y = \sin(2t)$ when $0 \le t \le \pi/2$.

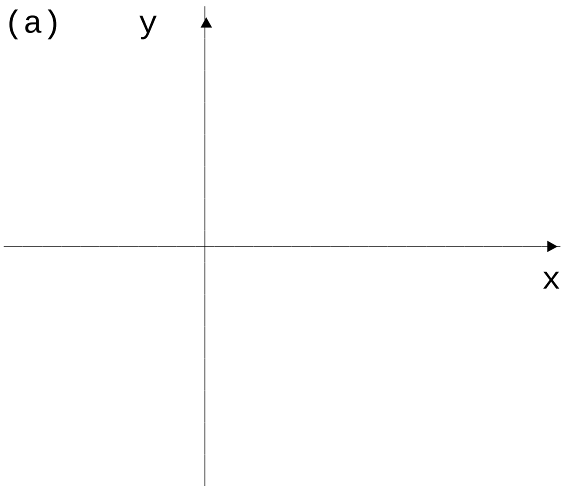
12. A spring exerts a force of 5 N when stretched 1 m beyond its natural length. How much work is required to stretch the spring 1.8 m beyond its natural length?

13. Evaluate the following integral.

$$\int x^2 e^x \, dx =$$

14. (a) Sketch the region in the 1st quadrant enclosed by the curves defined by y = sin x, y = 0, and x = π/2. Suppose the region is revolved around the y axis. (b) Using the method of cylindrical shells, compute the volume of the solid of revolution formed.

(a)



(b)

V =

15. Evaluate the following integral.

$$\int \frac{1}{(1+x^2)^{1/2}} \, dx =$$

16. Complete each trigonometric identity with an expression involving cos(2*x*).

(a) $\sin^2(x) =$

(b) $\cos^2(x) =$

17. Evaluate the integral.

$$\int e^{-x}\tan(e^{-x})\,dx =$$

18. Evaluate the integral.

$$\int \sec(4x)\,dx =$$

19. Evaluate the integral.

$$\int_1^e \ln(x)\,dx =$$

20. (a) Suppose that $n \geq 2$ is a positive integer. Show in detail how to derive the following reduction formula:

$$\int \cos^n(x)\,dx = \frac{\cos^{n-1}(x)\sin(x)}{n} + \frac{n-1}{n}\int \cos^{n-2}(x)\,dx$$

(b) Use the reduction formula in Part (a) to evaluate the following definite integral:

$$\int_0^{\pi/2} \cos^4(x)\,dx =$$