NAME: Em Toidi **MAC2312/Quiz-1**

READ ME FIRST: Show me all the magic very neatly on the page, for I do not read minds. Use correct notation when presenting your computations and arguments. Use complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer consists of all the magic transformations, do not "box" your final results. Eschew obfuscation.

1. (4 pts.) Express the following sum using sigma notation, but do not attempt to find its numerical value.

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5} = \sum_{k=1}^{5}\frac{(-1)^{k+1}}{k}$$

2. (4 pts.) Obtain the upper limit of summation and rewrite the summand in order to make the following equation true. **Do not attempt to evaluate the sum.**

$$\sum_{k=5}^{9} k\,2^{k+4} = \sum_{j=1}^{5}(j+4)2^{(j+4)+4} = \sum_{j=1}^{5}(j+4)2^{j+8}$$

since $j = k - 4$ is equivalent to $k = j + 4$.

3. (4 pts.) Evaluate the following sum in closed form.

$$\sum_{k=1}^{20}3^k = \sum_{j=0}^{19}3^{j+1} = \sum_{j=0}^{19}3\cdot 3^j = 3\sum_{j=0}^{19}3^j = 3\left[\frac{1-3^{20}}{1-3}\right] = \frac{3}{2}(3^{20}-1)$$

4. (4 pts.) Express the function of n in closed form and then find the limit.

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{5k}{n^2} = \lim_{n\to\infty}\frac{5}{n^2}\sum_{k=1}^{n}k = \lim_{n\to\infty}\left(\frac{5}{n^2}\right)\left(\frac{n(n+1)}{2}\right) = \lim_{n\to\infty}\left(\frac{5}{2}\right)\left(1+\frac{1}{n}\right) = \frac{5}{2}$$

5. (4 pts.) If the function f is continuous on [a,b], then the *net signed area* A between $y = f(x)$ and the interval [a,b] is defined by

$$A = \lim_{n\to\infty}\sum_{k=1}^{n}f(x_k^*)\Delta x.$$

Reveal all the details in obtaining the numerical value of the net signed area of $f(x) = x^2$ over the interval [0,1] using the definition above with x_k^* the right end point of each subinterval in the regular partition.

$$A = \lim_{n\to\infty}\sum_{k=1}^{n}f(x_k^*)\Delta x = \lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^2\frac{1}{n} = \lim_{n\to\infty}\left(\frac{1}{n^3}\right)\sum_{k=1}^{n}k^2$$

$$= \lim_{n\to\infty}\left(\frac{n(n+1)(2n+1)}{6n^3}\right) = \lim_{n\to\infty}\frac{1}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) = \frac{1}{3}$$

since $\Delta x = 1/n$ and $x_k = k/n$ for $k = 0, 1, \ldots, n$ are the end points of the intervals of the general regular partition.