NAME: Em Toidi **MAC2312/Quiz-2**

READ ME FIRST: Show me all the magic very neatly on the page, for I do not read minds. Use correct notation when presenting your computations and arguments. Use complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer consists of all the magic transformations, do not "box" your final results. Eschew obfuscation.

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1. (4 pts.) Express the following limit as a definite integral. Do not attempt to evaluate the integral.

$$L = \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} (\sin^2(x_k^*))\Delta x_k \ ; \quad a = 0,\ b = \pi/2.$$

$$L = \int_0^{\pi/2} \sin^2(x)\, dx.$$

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2. (4 pts.) (a) If $\int_{-1}^{2} f(x)\,dx = 5$ and $\int_{-1}^{2} g(x)\,dx = -3$, then

$$\int_{-1}^{2} [\, f(x) + 2g(x)\,]\, dx = \int_{-1}^{2} f(x)\,dx + 2\int_{-1}^{2} g(x)\,dx = 5 + (2)(-3) = -1$$

(b) If $\int_0^1 f(x)\,dx = -2$ and $\int_0^5 f(x)\,dx = 1$, then

$$\int_1^5 f(x)\,dx = \int_1^0 f(x)\,dx + \int_0^5 f(x)\,dx = 2 + 1 = 3.$$

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3. (6 pts.) Let $F(x) = \int_4^x (t^2+9)^{1/2}\, dt$. Find

(a) $F(4) = \int_4^4 (t^2+9)^{1/2}\, dt = 0$

(b) $F'(4) = 5$ *since* $F'(x) = (x^2+9)^{1/2}$.

(c) $F''(4) = 4/5$ *since* $F''(x) = x/(x^2+9)^{1/2}$.

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4. (3 pts.) Evaluate the given integral using Part 1 of the Fundamental Theorem of Calculus.

$$\int_0^{1/(2)^{1/2}} \frac{1}{(1-x^2)^{1/2}}\, dx = \sin^{-1}(x)\Big|_0^{1/(2)^{1/2}} = \sin^{-1}(1/(2)^{1/2}) - \sin^{-1}(0) = \pi/4.$$

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5. (3 pts.) Evaluate the following limit by intepreting it as a Riemann sum in which the given interval is divided into n subintervals of equal width.

$$L = \lim_{n \to +\infty} \sum_{k=1}^{n} \frac{\pi}{4n} \sec^2\!\left(\frac{\pi k}{4n}\right) \ ; \quad \left[0, \frac{\pi}{4}\right]$$

$$L = \int_0^{\pi/4} \sec^2(x)\,dx = \tan(x)\Big|_0^{\pi/4} = \tan(\pi/4) - \tan(0) = 1.$$