NAME: Em Toidi **MAC2312/Quiz-3**

READ ME FIRST: Show me all the magic very neatly on the page, for I do not read minds. Use correct notation when presenting your computations and arguments. Use complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer consists of all the magic transformations, do not "box" your final results. Eschew obfuscation.

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1. (6 pts.) A particle moves with acceleration $a(t) = 2$ m/s^2 along an s-axis and has velocity $v_0 = -2$ m/s at time $t = 0$. Write down the definite integrals that give the displacement and distance traveled during the time interval $0 \le t \le 2$ using all the information at hand, but do not attempt to evaluate the integrals.

(a) $v(t) = 2t - 2$

(b) Displacement $= \int_0^2 v(t)\,dt = \int_0^2 2t - 2\,dt$.

(c) Total_Distance $= \int_0^2 |v(t)|\,dt = \int_0^2 |2t-2|\,dt$

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2. (3 pts.) If the given integral is expressed as an equivalent integral in terms of the variable u using the substitution $u = 1 + e^x$ so that the equation below is true, what are the numerical values of the new limits of integration α and β, and what is the new integrand, $f(u)$??? Obtain these but do not attempt to evaluate the du integral.

$$\int_0^{\ln(3)} e^x(1+e^x)^{1/2}\,dx = \int_\alpha^\beta f(u)\,du$$

$\alpha = 2$ $\beta = 4$ $f(u) = u^{1/2}$

since $du = e^x dx$.

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3. (3 pts.) Give the definition of the function $\ln(x)$ in terms of a definite integral and give its domain and range. Label correctly.

$$\ln(x) = \int_1^x \frac{1}{t}\,dt$$

for $x > 0$. The domain of the natural log function is $(0,\infty)$, and its range is the whole real line, $(-\infty,\infty)$.

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4. (4 pts.) (a) Find the following derivative.

$$\frac{d}{dx}\left[\int_{\tan(x)}^3 \frac{t^2}{1+t^2}\,dt\right] = -\frac{d}{dx}\left[\int_3^{\tan(x)} \frac{t^2}{1+t^2}\,dt\right] = -\left[\frac{\tan^2(x)}{1+\tan^2(x)}\right]\sec^2(x) = -\tan^2(x).$$

(b) Evaluate the following limit.

$$L = \lim_{x\to\infty}\left(1+\frac{1}{2x}\right)^x = \lim_{u\to\infty}\left(1+\frac{1}{u}\right)^{u/2} = \lim_{u\to\infty}\left[\left(1+\frac{1}{u}\right)^u\right]^{1/2} = e^{1/2}.$$

using the substitution $u = 2x$.

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5. (4 pts.) (a) Write the solution to the following initial value problem in terms of a definite integral taken with respect to the variable t, so the differential denoting the variable of integration is dt. (b) Then reveal the true identity of y by evaluating the definite integral with respect to t.

$$\frac{dy}{dx} = \frac{1}{x\ln(x)} \quad \text{with} \quad y(e) = 1.$$

$$y(x) = 1 + \int_e^x \frac{1}{t\ln(t)}\,dt \qquad (a)$$

$$= 1 + \int_1^{\ln(x)} \frac{1}{u}\,du = 1 + \ln(\ln(x)) = \ln(\ln(x)) + 1,\ \textit{for } x > 1$$

using the u-substitution $u = \ln(t)$.