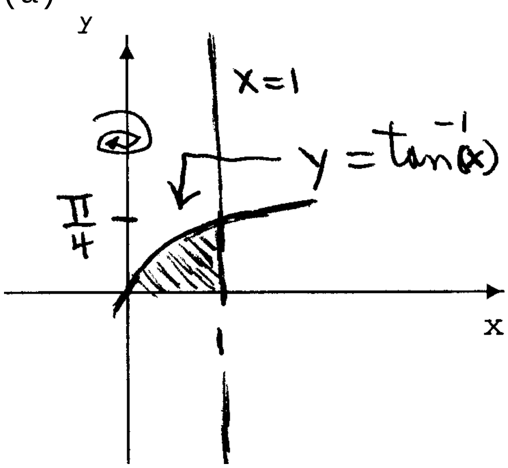
NAME: **Em Toidi** **MAC2312/Quiz-4**

READ ME FIRST: Show me all the magic very neatly on the page, for I do not read minds. Use correct notation when presenting your computations and arguments. Use complete sentences. Remember this: "=" denotes "equals" , "$\Rightarrow$" denotes "implies" , and "$\Leftrightarrow$" denotes "is equivalent to". Since the answer consists of all the magic transformations, do not "box" your final results. Eschew obfuscation.

1. (20 pts.: (a) is 5 points. Remaining parts are 3 points each.)
(a) Very carefully sketch the region R in the 1st quadrant enclosed by the curves defined by $y = \tan^{-1}(x)$, $y = 0$, and $x = 1$. Suppose the region is revolved around the y-axis. (b) Using the method of cylindrical shells, write down the definite integral dx used to compute the volume of the solid of revolution formed. **Don't evaluate the integral.** (c) Using the method of slicing [disks/washers here], write down the definite integral dy used to compute the same volume as in part (b). **Don't evaluate the integral.**

(a)



(b)

$$V = \int_0^1 2\pi x \tan^{-1}(x)\, dx$$

(c)

$$V = \int_0^{\pi/4} \pi(1)^2 - \pi \tan^2(y)\, dy$$

(d) Write down, but do not attempt to evaluate the definite integral whose numerical value gives the area of the region R above if one integrates with respect to y.

$$\text{Area} = \int_0^{\pi/4} 1 - \tan(y)\, dy$$

(e) Write down, but do not attempt to evaluate the definite integral whose numerical value gives the area of the region R above if one integrates with respect to x.

$$\text{Area} = \int_0^{1} \tan^{-1}(x)\, dx$$

(f) Using one of the integrals from either Part (d) or Part (e) above, compute the exact value of the area of the region R.

$$\text{Area} = \int_0^{\pi/4} 1 - \tan(y)\, dy = \frac{\pi}{4} - \int_0^{\pi/4} \frac{\sin(y)}{\cos(y)}\, dy$$

$$= \frac{\pi}{4} + \left(\ln|\cos(y)| \right)\Big|_0^{\pi/4} = \frac{\pi}{4} - \frac{1}{2}\ln(2).$$

Note: At this point in time, unless you have been exposed to integration by parts, (e) above is not accessible.