NAME: Em Toidi **MAC2312/Quiz-5**

READ ME FIRST: Show me all the magic very neatly on the page, for I do not read minds. Use correct notation when presenting your computations and arguments. Use complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer consists of all the magic transformations, do not "box" your final results. Eschew obfuscation.

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1. (4 pts.) Find the exact arc length of the curve $y = x^{3/2}$ over the interval from x = 1 to x = 4.

$$L = \int_1^4 \left(1 + \left(\frac{dy}{dx}\right)^2\right)^{1/2} dx = \int_1^4 \left(1 + \frac{9x}{4}\right)^{1/2} dx$$
$$= \int_{13/4}^{10} u^{1/2}\left(\frac{4}{9}\right) du, \quad using\ u = 1 + \frac{9x}{4}$$
$$= \left(\frac{8}{27}u^{3/2}\right)\Big|_{13/4}^{10} = \frac{8}{27}\left(10^{3/2} - \left(\frac{13}{4}\right)^{3/2}\right).$$

Note: The final answer here may be expressed in a variety of different, equivalent forms.

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2. (4 pts.) A spring exerts a force of 5 N when stretched 1 m beyond its natural length. How much work is required to stretch the spring 1.8 m beyond its natural length?

$$\text{Work} = \int_0^{9/5} F(x)\, dx = \int_0^{9/5} 5x\, dx = \left(\frac{5}{2}x^2\right)\Big|_0^{9/5} = \frac{81}{10}\ \text{J}$$ since (1)k = 5 implies

k = 5 N per meter using the standard Hooke's model.

Note: Using the decimal 1.8 leads to far messier arithmetic!!

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3. (12 pts.) Evaluate each of the following integrals.

(a) $$\int \tan^{-1}(3x)\, dx = x\tan^{-1}(3x) - \int \frac{3x}{1+9x^2}\, dx$$
$$= x\tan^{-1}(3x) - \frac{1}{6}\ln(1+9x^2) + C$$

using parts with $u = \tan^{-1}(3x)$ and $dv = 1\ dx$ followed by a substitution that you should be able to do in your head(s).

(b) $$\int x\ln(x)\, dx = \frac{x^2}{2}\ln(x) - \int \frac{x^2}{2}\cdot\frac{1}{x}\, dx = \frac{x^2}{2}\ln(x) - \int \frac{x}{2}\, dx$$
$$= \frac{x^2}{2}\ln(x) - \frac{x^2}{4} + C$$

using parts with $u = \ln(x)$ and $dv = x\ dx$.

(c) $$\int e^x \sin(x)\, dx = \sin(x)e^x - \int \cos(x)e^x\, dx$$
$$= \sin(x)e^x - \left[\cos(x)e^x - \int (-\sin(x))e^x\, dx\right]$$
$$= \sin(x)e^x - \cos(x)e^x - \int \sin(x)e^x\, dx$$

by integrating by parts twice, each time picking on e^x as the recognized derivative, i.e., choosing $dv = e^x\ dx$. Solving the linear equation above for the desired integral yields

$$\int e^x \sin(x)\, dx = \frac{1}{2}\left(\sin(x)e^x - \cos(x)e^x\right) + C.$$