NAME: Em Toidi **MAC2312/Quiz-6**

READ ME FIRST: Show me all the magic very neatly on the page, for I do not read minds. Use correct notation when presenting your computations and arguments. Use complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer consists of all the magic transformations, do not "box" your final results. Eschew obfuscation.

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1. (4 pts.) Using literal constants A, B, C, etc., write the form of the partial fraction decomposition for the proper fraction below. Do not attempt to obtain the actual numerical values of the constants A, B, C, etc. Be very careful here.

$$\frac{2x^2+4}{(x-1)^3(x^2+1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{Dx+E}{x^2+1} + \frac{Fx+G}{(x^2+1)^2}$$

(b) (4 pts.) If one were to integrate the rational function in part (a), one probably would encounter the integral below. Reveal, in detail, how to evaluate this integral.

$$\int \frac{1}{(x^2+1)^2}\,dx = \int \frac{\sec^2(\theta)d\theta}{(\sec^2(\theta))^2}, \quad \textit{when } \tan(\theta) = x,$$

$$= \int \cos^2(\theta)\,d\theta = \int \frac{1+\cos(2\theta)}{2}\,d\theta = \frac{\theta}{2} + \frac{\sin(\theta)\cos(\theta)}{2} + C$$

$$= \frac{\tan^{-1}(x)}{2} + \frac{x}{2(x^2+1)} + C.$$

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2. (12 pts.) Evaluate each of the following integrals.

(a) $\int \sin^2(t)\,dt = \int \frac{1-\cos(2t)}{2}\,dt = \frac{t}{2} - \frac{\sin(2t)}{4} = C$

by means of the usual trig or treat.

(b)

$$\int (1-x^2)^{1/2}\,dt = \int (\cos^2(\theta))^{1/2}\cos(\theta)\,d\theta$$
$$= \int \cos^2(\theta)\,d\theta$$
$$= \int \frac{1+\cos(2\theta)}{2}\,d\theta$$
$$= \frac{\theta}{2} + \frac{\sin(2\theta)}{4} + C$$
$$= \frac{\sin^{-1}(x)}{2} + \frac{x(1-x^2)^{1/2}}{2} + C$$

by using the trig substitution $x = \sin(\theta)$ *so that* $dx = \cos(\theta)d\theta$.

(c) $\int \frac{1}{x(x+1)}\,dx = \int \frac{1}{x} - \frac{1}{x+1}\,dx = \ln|x| - \ln|x+1| + C$

using an easy and obvious partial fraction decomposition.