NAME: Em Toidi **MAC2312/Quiz-7**

READ ME FIRST: Show me all the magic very neatly on the page, for I do not read minds. Use correct notation when presenting your computations and arguments. Use complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer consists of all the magic transformations, do not "box" your final results. Eschew obfuscation.

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1. (4 pts.) Consider the definite integral below. Write down the sum, S_4, used to approximate the value of the integral below if Simpson's Rule is used with $n = 4$. **Do not attempt to evaluate the sum. Be very careful.**

$$\int_1^3 x^{1/2}\, dx$$

$$S_4 = \frac{1}{6}\left(\left(\tfrac{2}{2}\right)^{1/2} + 4\left(\tfrac{3}{2}\right)^{1/2} + 2\left(\tfrac{4}{2}\right)^{1/2} + 4\left(\tfrac{5}{2}\right)^{1/2} + \left(\tfrac{6}{2}\right)^{1/2} \right)$$

since $\Delta x = \frac{1}{2}$, *and* $x_k = 1 + \frac{k}{2} = \frac{2+k}{2}$, *for* $k = 0,1,2,3,4$. are the points of the regular partition we need.

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2. (4 pts.) Make an appropriate u-substitution of the form $u = x^{1/n}$ or $u = (x + a)^{1/n}$ and then evaluate the integral.

$$\int \frac{dx}{x - x^{1/3}} = \int \frac{3u^2}{u^3 - u}du = \int \frac{3u}{u^2-1}du = \frac{3}{2}\ln|u^2-1| + C = \frac{3}{2}\ln|x^{2/3}-1| + C$$

using the substitution $u = x^{1/3}$ so that $x = u^3$ and $dx = 3u^2\, du$.

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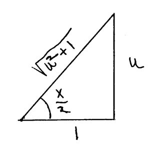
3. (4 pts.) Evaluate the following integral using the substitution $u = \tan(x/2)$.

$$\int \frac{1}{1-\cos(x)}\,dx = \int \frac{1}{1 - \left(\frac{1-u^2}{1+u^2}\right)} \cdot \frac{2}{1+u^2}\,du = \int \frac{1}{u^2}\,du = -u^{-1} + C$$

$$= -\frac{1}{\tan(x/2)} + C = -\cot\left(\frac{x}{2}\right) + C$$

To use the substitution, from the triangle to the right, you may read off that $\sin(x/2) = u/(u^2 + 1)^{1/2}$ and that $\cos(x/2) = 1/(u^2 + 1)^{1/2}$. Then $\sin(x) = 2u/(u^2 + 1)$ and $\cos(x) = (1 - u^2)/(u^2 + 1)$ using double angle formulae. And finally, $u = \tan(x/2)$ implies $x = 2\tan^{-1}(u)$, and $dx = (2/(u^2 + 1))du$. Now put all this to work to evaluate the integral.



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4. (8 pts.) Evaluate the integrals that converge.

(a)

$$\int_e^{+\infty} \frac{1}{x\ln^3(x)}\,dx = \lim_{m\to\infty}\int_e^m \frac{1}{x\ln^3(x)}\,dx = \lim_{m\to\infty}\int_1^{\ln(m)} \frac{1}{u^3}\,du$$

$$= \lim_{m\to\infty}\left(-\frac{1}{2}u^{-2}\right)\Big|_1^{\ln(m)} = \lim_{m\to\infty}\left[\frac{1}{2} - \frac{1}{2\ln^2(m)}\right] = \frac{1}{2}$$

by using the substitution $u = \ln(x)$.

(b)

$$\int_0^1 \frac{1}{(1-x^2)^{1/2}}\,dx = \lim_{m\to 1^-}\int_0^m \frac{1}{(1-x^2)^{1/2}}\,dx$$

$$= \lim_{m\to 1^-}\left[\sin^{-1}(m) - \sin^{-1}(0)\right] = \frac{\pi}{2}.$$