NAME: Em Toidi **MAC2312/Quiz-8**

READ ME FIRST: Show me all the magic very neatly on the page, for I do not read minds. Use correct notation when presenting your computations and arguments. Use complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer consists of all the magic transformations, do not "box" your final results. Eschew obfuscation.

1. (4 pts.) Find the general term of the sequence, starting with $n = 1$, determine whether the sequence converges, and if so, find its limit.

$$\frac{1}{3}, -\frac{1}{9}, \frac{1}{27}, -\frac{1}{81}, \ldots .$$

$$a_n = \frac{(-1)^{n+1}}{3^n} \ \textit{for } n \geq 1. \ \textit{Thus, } \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{(-1)^{n+1}}{3^n} = 0.$$

Observe we are dealing with a *sequence*. Note the commas. If necessary, the squeeze theorem for sequences may be used to see the limit by using the following inequality:

$$-\frac{1}{3^n} \leq \frac{(-1)^{n+1}}{3^n} \leq \frac{1}{3^n} \ \textit{for } \ n \geq 1.$$

2. (2 pts.) Express the repeating decimal as a fraction. $0.4444\ldots = \dfrac{4}{9}$

either by using the "high school" method or summing an appropriate geometric series.

3. (2 pts.) [Complete the following.] The harmonic series has the form $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k}$

What is the sum of the harmonic series? The harmonic series does not converge, and thus, has no sum.

4. (8 pts.) Determine whether the series converges, and if so, find its sum.

(*a*) $$\sum_{k=1}^{\infty} \left(-\frac{3}{4}\right)^{k-1} = \sum_{j=0}^{\infty} \left(-\frac{3}{4}\right)^{j} = \frac{1}{1 - \left(-\frac{3}{4}\right)} = \frac{4}{7}.$$

(*b*) $$\sum_{k=1}^{\infty} \frac{1}{(k+2)(k+3)} = \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{1}{k+2} - \frac{1}{k+3}\right) = \lim_{n\to\infty} \left(\frac{1}{3} - \frac{1}{n+3}\right) = \frac{1}{3}.$$

5. (4 pts.) Find all values of x for which the series converges, and find the sum of the series for those values of x.

$$\frac{1}{x^2} + \frac{2}{x^3} + \frac{4}{x^4} + \frac{8}{x^5} + \frac{16}{x^6} + \ldots$$

Evidently, this is a geometric series. Writing it using sigma notation makes things easy.

Thus, $$\sum_{k=0}^{\infty} \frac{2^k}{x^{k+2}} = \sum_{k=0}^{\infty} \frac{1}{x^2}\left(\frac{2}{x}\right)^k = \left(\frac{1}{x^2}\right)\left(\frac{1}{1 - \frac{2}{x}}\right) = \frac{1}{x^2 - 2x}$$ provided that we have

$$\left|\frac{2}{x}\right| < 1, \quad \textit{or} \quad 2 < |x|, \quad \textit{or} \quad x < -2 \ \textit{or} \ 2 < x.$$