NAME: **Em Toidi** **MAC2312/Quiz-9**

READ ME FIRST: Show me all the magic very neatly on the page, for I do not read minds. Use correct notation when presenting your computations and arguments. Use complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer consists of all the magic transformations, do not "box" your final results. Eschew obfuscation.

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1. (4 pts.) Use ratio test to determine whether the series converges, If the test is inconclusive, say so.

$\sum_{k=1}^{\infty} \frac{3^k}{k!}$ Since $\rho = \lim_{k \to \infty} \frac{3^{k+1}/(k+1)!}{3^k/k!} = \lim_{k \to \infty} \frac{3}{k+1} = 0$, and $\rho < 1$, ratio test implies that the given series converges.

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2. (4 pts.) Use root test to determine whether the series converges. If the test is inconclusive, say so.

$\sum_{k=1}^{\infty} \left(\frac{3k+2}{2k-1}\right)^k$ Since $\rho = \lim_{k \to \infty} \left[\left(\frac{3k+2}{2k-1}\right)^k\right]^{1/k} = \lim_{k \to \infty} \frac{3k+2}{2k-1} = \frac{3}{2}$, and $\rho > 1$, root test implies that the given series diverges.

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3. (4 pts.) Use comparison test to show the following series diverges.

$\sum_{k=1}^{\infty} \frac{\ln(k)}{k}$ Plainly, for $k \geq 3$ we have $\ln(k)/k \geq 1/k$. Since the harmonic series $\sum k^{-1}$ diverges, all its tail series diverge. Comparison of the tail series that begin with $k = 3$ allows us to conclude that the tail of the original series that begins with $k = 3$ also diverges. Hence the given series diverges.

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4. (4 pts.) Apply the divergence test and state what it tells you about each of the following series.

(*a*) $\sum_{k=1}^{\infty} \cos(k\pi)$ Since $\lim_{k \to \infty} \cos(k\pi) = \lim_{k \to \infty} (-1)^k$ does not exist, the limit of the sequence of terms is not equal to zero. Thus, divergence test implies that (a) diverges.

(*b*) $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$ Since $\lim_{k \to \infty} \frac{1}{\sqrt{k}} = 0$, divergence test provides no information concerning the convergence of (b).

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5. (4 pts.) Confirm that the integral test is applicable and then use it to determine whether the following series converges:

$\sum_{k=1}^{\infty} \frac{1}{1+9k^2}$ Let $f(x) = (1+9x^2)^{-1}$ for $x \geq 1$. Clearly f is a positive continuous function. Since $f'(x) = -18x(1+9x^2)^{-2} < 0$ for $x > 1$, f is decreasing on $[1,\infty)$. Evidently the terms of the series are given by $f(k)$. Thus we may apply integral test. Since we have

$$\int_1^{\infty} f(x)\,dx = \int_1^{\infty} \frac{1}{1+9x^2}\,dx = \lim_{b \to \infty} \frac{1}{3}\int_1^b \frac{3}{1+(3x)^2}\,dx$$
$$= \lim_{b \to \infty} \left[\frac{1}{3}\tan^{-1}(3b) - \frac{1}{3}\tan^{-1}(3)\right]$$
$$= \frac{\pi - 2\tan^{-1}(3)}{6},$$

integral test implies that the given series converges.