**NAME: Em Toidi** **MAC2312/Quiz-10**

READ ME FIRST: Show me all the magic very neatly on the page, for I do not read minds. Eschew obfuscation.

**1. (4 pts.)** Classify the given series as absolutely convergent, conditionally convergent, or divergent. Warning: For full credit, proof is required.

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}$$

Since

$$\sum_{k=1}^{\infty} |(-1)^k/\sqrt{k}| = \sum_{k=1}^{\infty} 1/\sqrt{k}$$

is a divergent p-series, the given series is not absolutely convergent. The given series is, however, an alternating series that satisfies the hypotheses of the alternating series test. Thus, it converges, but not absolutely. Hence the given series is conditionally convergent.

**2. (4 pts.)** The series below satisfies the hypotheses of the alternating series test. Find a value of $n$ for which the nth partial sum is ensured to approximate the sum of the series to two decimal places. Warning: For full credit, proof is required.

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k}}$$

From the error bound for A.S.T., if $s$ is the sum, for $n \geq 1$ we have

$$\left| s - \sum_{k=1}^{n} \frac{(-1)^{k+1}}{\sqrt{k}} \right| < \frac{1}{\sqrt{n+1}}.$$

From this, to obtain the desired accuracy, it suffices to find a positive integer $n$ so that $1/(n+1)^{1/2} \leq (1/2)(10^{-2})$. This last inequality is actually equivalent to $n \geq 39999$. Thus, $n = 39999$ will do the job.

**3. (8 pts.)** [Complete the following.] (a) If $f(2) = 3$, $f'(2) = -4$, and $f''(2) = 10$, then the second Taylor polynomial for $f$ about x = 2 is

$$p_2(x) = f(2) + \frac{f'(2)}{1!}(x-2) + \frac{f''(2)}{2!}(x-2)^2 = 3 - 4(x-2) + 5(x-2)^2.$$

(b) If $f$ has derivatives of all orders at $x_0$, then the Taylor series for $f$ at $x = x_0$ is defined to be

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$$

(c) If a function $f$ has an $n$th Taylor polynomial $p_n(x)$ about $x = x_0$, then the $n$th remainder $R_n(x)$ is defined by

$$R_n(x) = f(x) - p_n(x)$$

(d) Suppose the function $f$ can be differentiated five times on the interval I containing $x_0 = 2$ and that $|f^{(5)}(x)| \leq 20$ for all $x$ in $I$. Then, for all $x$ in $I$,

$$|R_4(x)| \leq \frac{20}{5!}|x-2|^5$$

**4. (4 pts.)** Consider $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{k\,10^k}(x-1)^k$. From ratio test for absolute convergence,

since $\displaystyle\lim_{k\to\infty}\left|\frac{u_{k+1}}{u_k}\right| = \frac{1}{10}|x-1|$, the radius of convergence is R = 10.

Substitution of $x = -9$ yields $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{2k}}{k} = \sum_{k=1}^{\infty} \frac{1}{k}$, and substitution of $x = 11$ yields $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{k}$.

Consequently, the interval of convergence is $I = (-9, 11]$.