
READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page, for I do not read minds. Eschew obfuscation.*

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1. (5 pts.) (a) On the first line write the expression using a definite integral that would be used to compute the average value of f(x) = sin(x) over the interval [π/3,π]. (b) Then below evaluate the expression to obtain the numerical value of the average.

(a) f_{AVE} =

(b) =

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2. (5 pts.) Using a complete sentence and appropriate notation, state precisely the First Part of the Fundamental Theorem of Calculus. [This is sometimes called the *Evaluation Theorem*.]

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3. (5 pts.) Express the following sum using sigma notation, but do not attempt to find its numerical value.

$$-\frac{1}{2^2}+\frac{1}{4^2}-\frac{1}{6^2}+\frac{1}{8^2}-\frac{1}{10^2}+\frac{1}{12^2} =$$

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4. (5 pts.) Obtain the upper limit of summation and rewrite the summand in order to make the following equation true. **Do not attempt to evaluate the sum.**

$$\sum_{k=0}^{45} 2^{3k} = \sum_{j=8}$$

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5. (5 pts.) Express the following limit as a definite integral. Do not attempt to evaluate the integral.

$$\mathrm{L} = \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} (2\sin(e^{x_k^*}))\Delta x_k \;; \quad a = -\pi,\ b = 2.$$

L =

6. (5 pts.) Using complete sentences and appropriate notation, state the Second Part of the Fundamental Theorem of Calculus.

7. (10 pts.) Find each of the following derivatives.

(*a*) $\frac{d}{dx}\left[\int_{x}^{\pi} 4\cos(e^{t^3})\ dt \right] =$

(*b*) $\frac{d}{dx}\left[\int_{1}^{x^2} \cos^3(t)\ dt \right] =$

8. (5 pts.) (a) (3 pts.) Give the definition of the function ln(x) in terms of a definite integral and give its domain and range. Label correctly. (**Hint**: To start, complete the sentence,"ln(x) = ...")

(b) (2 pts.) Given *a* and *b* are positive numbers with ln(*a*) = 4 and ln(*b*) = -3, evaluate the following integral.

$\int_{b^2}^{a} \frac{1}{t}\ dt =$

9. (5 pts.) Write the solution to the following initial value problem in terms of a definite integral taken with respect to the variable **t**, so the differential denoting the variable of integration is ***dt***. DO NOT ATTEMPT TO EVALUATE THE DEFINITE INTEGRAL IN *t* THAT YOU OBTAIN.

$$\frac{dy}{dx} = 4\sin(x^2), \quad y(\pi) = 1.$$

$y(x) =$

10. (5 pts.) Evaluate the definite integral below by expressing it in terms of u correctly and then evaluating the resulting integral using a formula from geometry.

$$\int_{\tan^{-1}(-9)}^{0} \sqrt{81-\tan^2(x)}\, 4\sec^2(x)\ dx \ ; \ u = \tan(x)$$

$$\int_{\tan^{-1}(-9)}^{0} \sqrt{81-\tan^2(x)}\, 4\sec^2(x)\ dx \ =$$

11. (10 pts.) Write each of the following two sums in closed form.

(a) $\displaystyle\sum_{k=0}^{49}\left(\frac{1}{2^k}\right) =$

(b) $\displaystyle\sum_{k=1}^{n}\left(\frac{1}{k+2} - \frac{1}{k+3}\right) =$

12. (10 pts.) If the function *f* is continuous on [a,b], then the *net signed area* A between *y* = *f*(*x*) and the interval [a,b] is defined by

$$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k^*)\,\Delta x.$$

Reveal all the details in computing the numerical value of the net signed area of $f(x) = 16x^3$ over the interval [0,1] using only the definition above with

$$x_k^*$$

the right end point of each subinterval in the regular partition. *Do not use the Fundamental Theorem, Part 1.*

13. (10 pts.) A particle moves with a velocity of $v(t) = 4 - 2t$ along an s-axis. Find the displacement and total distance traveled over the time interval [0,3].

Displacement =

Total_Distance =

14. (5 pts.) Evaluate the following limit:

$$L = \lim_{x \to \infty} \left(1 + \frac{\ln(3)}{x}\right)^x =$$

15. (10 pts.) Let the function g be defined by the equation

$$g(x) = \int_0^x \tan^{-1}(t)\, dt + \frac{\pi}{2}x$$

for x ε (-∞ , ∞). Then

(a) $g(0) =$

(b) $g'(0) =$

(c) $g''(0) =$

(d) Determine the open intervals where g is increasing or decreasing. Be specific.

(e) Determine the open intervals where g is concave up or concave down. Be specific.

Silly 10 Point Bonus: State the *Mean-Value Theorem for Integrals* and then use it to prove *Part 2 of the Fundamental Theorem of Calculus*.
[Say where your work is, for it won't fit here.]