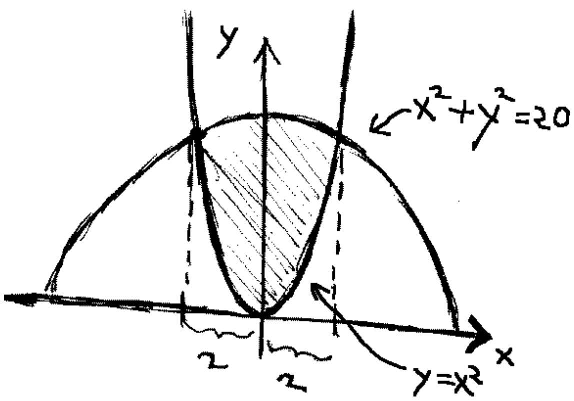
READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Eschew obfuscation. Show me all the magic on the page.*

1. (25 pts.)
The region R bounded between the curves $x^2 + y^2 = 20$ and $y = x^2$ is sketched below for your convenience. *The sketch does not have all the information you need for this problem!!*



(a) Write down, but do not attempt to evaluate the definite integral whose numerical value gives the area of the region R if one integrates with respect to x so the differential in the integral is dx.

Area =

(b) Write down, but do not attempt to evaluate the sum of definite integrals whose numerical value gives the area of the region R if one integrates with respect to y so the differential in the integral is dy.

Area =

(c) A first step to evaluate the definite integral in part (a) above is to perform a suitable trigonometric substitution. Explicitly give the substitution and provide the definite integral with respect to θ that results, but do not attempt to evaluate the $d\theta$ integral you have obtained.

Area =

(d) Write down, but do not attempt to evaluate, the definite integral that provides the numerical value of the arc-length of the lower curve above, $y = x^2$ from $x = -2$ to $x = 2$.

Length =

(e) A first step to evaluate the definite integral in part (d) above is to perform a suitable trigonometric substitution. Explicitly give the substitution and provide the definite integral with respect to θ that results, but do not attempt to evaluate the $d\theta$ integral you have obtained.

Length =

2. (5 pts.) Suppose that $n \geq 2$ is a positive integer. By using integration by parts and an appropriate trigonometric identity, show in detail how to derive the following reduction formula:

$$\int \cos^n(x)\ dx = \frac{\cos^{n-1}(x)\sin(x)}{n} + \frac{n-1}{n}\int \cos^{n-2}(x)\ dx$$

3. (20 pts.) (a) (5 pts.) Using literal constants A, B, C, etc., write the form of the partial fraction decomposition for the proper fraction below. Do not attempt to obtain the actual numerical values of the constants A, B, C, etc. Be very careful here.

$$\frac{3x^2+5}{x(x-2)(x^2+1)^3} =$$

(b) (5 pts.) Obtain the numerical values of the literal constants *A*, *B*, and *C* in the partial fraction decomposition below.

$$\frac{x-2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

(c) (10 pts.) If one were to integrate the rational function in part (a), one might also encounter the integral below. Evaluate the integral below.

$$\int \frac{1}{(x^2+1)^3}\ dx =$$

4. (50 pts.) Evaluate each of the following antiderivatives or definite integrals. Give exact values for definite integrals. Be careful, for some of the definite integrals are improper. To get any credit, they must be handled correctly!!! [5 pts./part]

(a) $\displaystyle\int_0^{\sqrt{\ln(7)}} 6x\,e^{x^2}\,dx =$

(b) $\displaystyle\int_0^{\infty} x\,e^{-x}\,dx =$

(c) $\displaystyle\int_0^{\pi} 16\sin^2(t)\,dt =$

(d) $\displaystyle\int_0^{\pi/4} 2\tan(2x)\,dx =$

(e) $\displaystyle\int \frac{8t}{\sqrt{1+4t^2}}\,dt =$

4. (Cont.) Evaluate each of the following antiderivatives or definite integrals. Give exact values for definite integrals. Be careful, for some of the definite integrals are improper. To get any credit, they must be handled correctly!!! [5 pts./part]

(f) $\int \frac{\sin^3(x)}{\cos(x)}\, dx =$

(g) $\int_0^1 \frac{2x+1}{x^2+1}\, dx =$

(h) $\int x^2 \cos(x)\, dx =$

(i) $\int \frac{4}{x^2-x}\, dx =$

(j) $\int \frac{2x^2+1}{x+2}\, dx =$

Silly 10 Point Bonus: (a) Prove that

(*) $$\frac{1}{t} \geq \frac{4}{3} - \frac{4t}{9}$$

for every t ε [1,2]. (b) Using (a) to compare a couple of integrals, prove ln(2) > 2/3. //Say where your work is, for there isn't room here.