READ ME FIRST: Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page. Eschew obfuscation.

1. (4 pts.) Find the general term of the sequence, starting with $n = 1$, determine whether the sequence converges, and if so, find its limit.

$$\left(1 + \frac{1}{1}\right)^3, \left(1 + \frac{1}{2}\right)^5, \left(1 + \frac{1}{3}\right)^7, \left(1 + \frac{1}{4}\right)^9, \left(1 + \frac{1}{5}\right)^{11}, \ldots .$$

2. (4 pts.) Express the repeating decimal as a fraction, more specifically as a quotient of positive integers. [The fraction does not have to be in lowest terms.]

$0.121121\underline{121}\ldots =$

3. (4 pts) Using complete sentences and appropriate notation, give the precise ε - N definition of

$$\lim_{n \to \infty} a_n = L.$$

4. (8 pts.) Complete the following by supplying the summand and filling in the blanks appropriately.

(a) A p-series is a series of the form

$$\sum_{k=1}^{\infty}$$

This series diverges if _______________ and this series converges if

_______________.

(b) A geometric series is a series of the form

$$\sum_{k=0}^{\infty}$$

This series diverges if _______________ and this series converges if

_______________.

5. (4 pts.) Give the precise mathematical definition of the sum of an infinite series,

$$\sum_{k=1}^{\infty} a_k$$

6. (8 pts.) Determine whether the series converges, and if so, find its sum.

(a) $$\sum_{k=1}^{\infty} \left(-\frac{3}{2}\right)^{k+2}$$

(b) $$\sum_{k=1}^{\infty} \left[\frac{1}{\ln(k+2)} - \frac{1}{\ln(k+3)}\right]$$

7. (4 pts.) Find all values of x for which the series converges, and find the sum of the series for those values of x.

$$x^2 + \frac{x^3}{5} + \frac{x^4}{25} + \frac{x^5}{125} + \frac{x^6}{625} + ..$$

8. (4 pts.) Apply the divergence test and state what it tells you about each of the following series.

(a) $$\sum_{k=1}^{\infty} \frac{k}{k+1}$$

(b) $$\sum_{k=1}^{\infty} \frac{\ln(k)}{\sqrt{k}}$$

9. (6 pts.) Let $f(x) = \sin(\pi x)$.
Obtain the 3rd Taylor polynomial of $f(x)$ about $x_0 = 1$.

$p_3(x) =$

10. (4 pts.) Use root test to determine whether the series converges. If the test is inconclusive, say so.

$$\sum_{k=1}^{\infty} \left(\frac{1}{2} + \frac{2}{k} \right)^k$$

11. (6 pts.) Classify each of the following series as absolutely convergent (AC), conditionally convergent (CC), divergent (D), or none of the preceding, (N). Circle the letters corresponding to your choice. (No explicit proof is needed.)

(a) $\sum_{k=1}^{\infty} \frac{(-1)^{k-1}\sqrt{k}}{k^{1/4}+1}$ (AC) (CC) (D) (N)

(b) $\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{(2k)!}$ (AC) (CC) (D) (N)

(c) $\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{\sqrt{k}+4}$ (AC) (CC) (D) (N).

12. (4 pts.) Confirm that the integral test is applicable, and then use it to determine whether the following series converges:

$$\sum_{k=1}^{\infty} \frac{1}{k^2+1}$$

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13. (4 pts.) Consider $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}\,10^k}(x-1)^k$. From ratio test for absolute convergence, since $\lim_{k \to \infty} \left| \frac{u_{k+1}}{u_k} \right| = \frac{1}{10}|x-1|$. ,the radius of convergence is R =

Substitution of x = -9 yields $\sum_{k=1}^{\infty} \frac{(-1)^{2k}}{\sqrt{k}} = \sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$, and substitution of x = 11 yields $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}$ Consequently, the interval of convergence is I =

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14. (4 pts.) Use ratio test to determine whether the series converges. If the test is inconclusive, say so.

$$\sum_{k=1}^{\infty} \frac{5}{k^2}$$

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15. (4 pts.) Use comparison test to show the following series diverges.

$$\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k+4}$$

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16. (4 pts.) $\ln(1.1) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k10^k}$

Use the error estimate from alternating series test to find an upper bound on the absolute error when the finite sum

$$s_5 = \sum_{k=1}^{5} \frac{(-1)^{k+1}}{k10^k}$$

is used to approximate ln(1.1).

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17. (4 pts.) Complete the following appropriately.

(a) If f has derivatives of all orders at x_0, then the Taylor series for f at $x = x_0$ is defined to be

$$\sum_{k=0}^{\infty}$$

(b) Suppose the function f can be differentiated five times on the interval I containing $x_0 = 2$ and that $|f^{(5)}(x)| \le 20$ for all x in I. Then, for all x in I,

$$|R_4(x)| \le$$

18. (4 pts.) Determine whether the following sequence is eventually increasing or eventually decreasing or neither:

$$\left\{ \frac{n!}{10^n} \right\}_{n=1}^{\infty}$$

19. (8 pts.) (a) Suppose $f(x) = \sum_{k=0}^{\infty} a_k (x - x_0)^k$ and $\lim\limits_{k \to \infty} \dfrac{|a_{k+1}|}{|a_k|} = 0$.

What is the domain of the power series function f ?

(b) Suppose $g(x) = \sum_{k=0}^{\infty} b_k (x - x_0)^k$ and $\lim\limits_{k \to \infty} \dfrac{|b_{k+1}|}{|b_k|} = +\infty$. What is the domain of the power series function g ?

20. (4 pts.) Let the sequence $\{a_n\}$ be defined recursively by

$$a_1 = 1, \quad and \quad a_{n+1} = \frac{1}{2}\left(a_n + \frac{25}{a_n}\right)$$

for $n \geq 1$. Assuming the sequence converges, find its limit L.

21. (4 pts.) Find a positive integer N so that if $n \geq N$, then

$$\left| \frac{5n}{n+2} - 5 \right| < \left(\frac{1}{2}\right) 10^{-3}$$

and prove it provides the desired error bound.

Silly 10 Point Bonus: Prove that the sequence in Problem 20 is bounded below by $m = 0$ and eventually decreasing. A key piece is the minimum of

$$g(x) = \frac{1}{2}\left(x + \frac{25}{x}\right) \text{ for } x > 0.$$

What does this now imply about the sequence??
[Say where your work is, for it won't fit here.]