

STUDENT NUMBER:

EXAM NUMBER:

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Read Me First:

*Read each problem carefully and do exactly what is requested. Full credit will be awarded only if you show all your work neatly, and it is correct. Use complete sentences and use notation correctly. Remember that what is illegible or incomprehensible is worthless. Communicate. Eschew obfuscation. Good Luck! [Total Points Possible: 120]*

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1. (20 pts.) (a) Using a complete sentence, state the first part of the Fundamental Theorem of Calculus, the evaluation theorem.

(b) Using complete sentences, state the second part of the Fundamental Theorem of Calculus.

(c) Compute  $g'(x)$  when  $g(x)$  is defined by the following equation.

$$g(x) = \int_0^x \tan^{-1}(t) + 29 \, dt + e^{x^2}$$

$$g'(x) =$$

(d) Give the definition of the function  $\ln(x)$  in terms of a definite integral, and give its domain and range. Label correctly.

(e) Write the solution to the following initial value problem in terms of a definite integral taken with respect to the variable  $t$ , so the differential denoting the variable of integration is  $dt$ . Then reveal an alternative identity of  $y$  by actually evaluating the definite integral with respect to  $t$ .

$$\frac{dy}{dx} = \frac{6 \ln^5(x)}{x} \quad \text{with} \quad y(e) = 59.$$

$$y(x) =$$

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2. (5 pts.) Suppose that

$$f(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k}} 5^k (x-1)^k$$

Find the radius of convergence and the interval of convergence of  $f$ .

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3. (10 pts.) Each of the following power series functions is the Maclaurin series of some well-known function. In each case, (i) identify the function, and (ii) provide the interval in which the series actually converges to the function.

(a)  $\sum_{k=0}^{\infty} x^k =$

(b)  $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} =$

(c)  $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1} =$

(d)  $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} =$

(e)  $\sum_{k=0}^{\infty} \frac{(-1)^k x^{k+1}}{k+1} =$

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4. (5 pts.) Let  $f(x) = \sin(\pi x)$ .

Obtain the 3<sup>rd</sup> Taylor polynomial of  $f(x)$  about  $x_0 = -2$ .

$p_3(x) =$

5. (8 pts.) (a)  $\ln(1.1) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k10^k}$

Use the error estimate from alternating series test to determine a specific value of  $n \geq 1$  so that the partial sum  $s_n$  approximates  $\ln(1.1)$  to 6 decimal places, where, of course,

$$S_n = \sum_{k=1}^n \frac{(-1)^{k+1}}{k10^k}$$

(b) Use the *Remainder Estimation Theorem* to obtain an interval containing  $x = 0$  in which  $f(x) = \sin(x)$  can be approximated to two decimal place accuracy by

$$p(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

6. (6 pts.) Suppose a spring has a natural length of 1 foot and a force of 15 pounds is required to compress the spring to a length of 9 inches. How much work is done in stretching this spring from its natural length to a length of 18 inches? [Assume Hooke's Law is valid.]

7. (6 pts.) Here are three convergent infinite series that should be very easy to sum up at this stage. Provide the value of each sum.

(a)  $\sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{7\pi}{6}\right)^{2k+1}}{(2k+1)!} =$

(b)  $\sum_{k=0}^{\infty} \left(\frac{29}{2}\right) \left(\frac{1}{2}\right)^k =$

(c)  $\sum_{k=0}^{\infty} \frac{(\ln(59))^k}{k!} =$

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8. (20 pts.) Here are five easy antiderivatives to evaluate.

(a)  $\int 2 \sin^2\left(\frac{x}{2}\right) dx =$

(b)  $\int \ln(x+8) dx =$

(c)  $\int \frac{4}{\cos(2t)} dt =$

(d)  $\int (4x-2)e^x dx =$

(e)  $\int \frac{\sec^3(\theta)}{\tan(\theta)} d\theta =$

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9. (10 pts.) A first step in evaluating each of the definite integrals below is to perform a suitable trigonometric substitution. *Explicitly give the substitution and provide the definite integral with respect to  $\theta$  that results, but do not attempt to evaluate the  $d\theta$  integral you have obtained.*

(a)  $\int_1^{\sqrt{2}} \frac{1}{x^2 \sqrt{4-x^2}} dx =$

(b)  $\int_1^2 \frac{\sqrt{x^2+1}}{x} dx =$

10. (10 pts.) (a) (2 pts.) Using literal constants  $A$ ,  $B$ ,  $C$ , etc., write the form of the partial fraction decomposition for the proper fraction below. Do not attempt to obtain the actual numerical values of the constants  $A$ ,  $B$ ,  $C$ , etc.

$$\frac{59x^2-29}{x(x-2)^2(x^2+5)^2} =$$

(b) (3 pts.) Obtain the numerical values of the literal constants  $A$ ,  $B$ , and  $C$  in the partial fraction decomposition given below.

$$\frac{x^2+4x+3}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

(c) (5 pts.) Find the following integral:

$$\int \frac{x^2+4x+3}{x(x^2+1)} dx =$$

11. (10 pts.) Suppose

$$f(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k 8^k} (x-2)^k$$

(a) By using sigma notation and term-by-term differentiation as done in class, obtain a power series for  $f'(x)$ .

$$f'(x) =$$

(b) By using sigma notation and integrating term-by-term as done in class, obtain an infinite series whose sum has the same numerical value as that of the following definite integral. [We are working with the power series  $f$  above.]

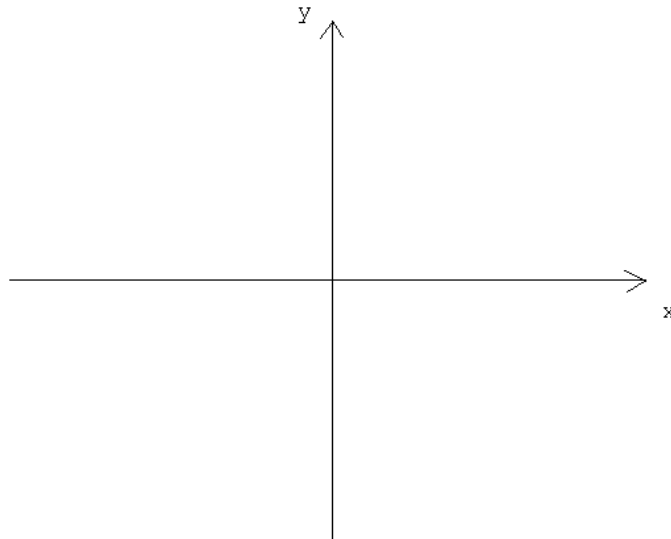
$$\int_2^7 f(x) dx =$$

12, (6 pts.) Sketch the curve  $r = 4\sin(\theta)$  in polar coordinates. Do this as follows: (a) Carefully sketch the auxiliary curve, a rectangular graph, on the  $r, \theta$ -coordinate system provided. (b) Then translate this graph to the polar one.

(a)



(b) [Think of polar coordinates lying over the  $x, y$  axes below.]



(c) Write down, but do not attempt to evaluate a definite integral with respect to  $\theta$  that provides the numerical value of the area enclosed by the polar curve given in Part (b) above.

Area =

13. (4 pts.) Evaluate the following integral:

$$\int_0^{\pi} \sqrt{1 - \cos(x)} \, dx =$$

Silly 10 point bonus: You may do exactly one of the following:

(a) State the Mean-Value Theorem for Integrals and use it to prove the second part of the Fundamental Theorem of Calculus, or

(b) Identify the function  $f$  given by the power series in Problem 11 above.

State which bonus you are attempting and where your work is.