STUDENT #: EXAM #:

Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations. Write using complete sentences. Remember this: "=" denotes "equals" , "$\Rightarrow$" denotes "implies" , and "$\Leftrightarrow$" denotes "is equivalent to". Vector objects must be denoted by using arrows. Since the answer really consists of all the magic transformations, do not "box" your final results. Show me all the magic on the page neatly.

1. (15 pts.) Obtain an equation for the plane that contains the three points (-1,0,1), (0,2,0), and (0,0,3).

2. (15 pts.) Obtain an arc-length parameterization for the curve $\mathbf{r}(t) = \langle 3t, 4\cos(t), 4\sin(t) \rangle$ in terms of the initial point $(3\pi, -4, 0)$. Rather than overloading the symbol **r**, write this new parameterization as $\mathbf{R}(s)$. If you move along the curve using the parameterization given by **R**, what's your speed?

3. (15 pts.) A particle, starting at (0,0) moves to the point (1,1) along the curve $y = x^4$ and then returns to (0,0) along the line $y = x$. Use Green's Theorem to compute the work done on the particle by the force field $\mathbf{F}(x,y) = \langle 6xy, 3x^2 + 8x \rangle$.

4. (15 pts.) Write down but do not attempt to evaluate the iterated triple integrals in (a) rectangular, (b) cylindrical, and (c) spherical coordinates that would be used to compute the volume enclosed between the xy-plane and the lower half of the sphere defined by $x^2 + y^2 + z^2 = (a_0)^2$, where $a_0 > 0$ is a constant.
[*Lower* means $z \leq 0$.]

(a) V =

(b) V =

(c) V =

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5. (15 pts.) (a) Find the unit vectors **T**(t) and **N**(t) and the curvature κ(t) for the helix defined by

$$\mathbf{r}(t) = \langle 5\cdot\cos(t),\ 12t,\ 5\cdot\sin(t)\rangle.$$

(b) Locate the center $C(x_0,y_0,z_0)$ of the circle of curvature when $t = \pi/2$.

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6. (15 pts.) Convert the double integral

$$\int_{-3}^{0}\int_{0}^{(9-x^2)^{1/2}} xy\ dy\,dx$$

to an equivalent integral in polar coordinates, and then evaluate the integral you obtain. [A picture of the region helps?]

$$\int_{-3}^{0}\int_{0}^{(9-x^2)^{1/2}} xy\ dy\,dx =$$

7. (15 pts.) Determine the maximum and minimum values of $f(x,y) = x^2 + y^2$ when (x,y) lies in the closed disk defined by the inequality $(x - 1)^2 + (y - 2)^2 \leq 1$. Analyze f on the boundary by using Lagrange Multipliers.

8. (15 pts.)

Let $f(x,y) = 6y^2 - 8x^2$. Compute the gradient of f at (1,-1), and then use it to compute $D_{\mathbf{u}}f(1,-1)$, where **u** is the unit vector in the same direction as **v** = <-2,5>.

$\nabla f(x,y) =$

$\nabla f(1,-1) =$

u =

$D_{\mathbf{u}}f(1,-1) =$

9. (15 pts.) Evaluate the following line integral, where C is the path from (1,-1) to (1,1) along the curve $x = y^2$. [Hint: The field is not conservative. The path is not closed. Parameterize C first. A picture might help.]

$$\int_C (y - x)\,dx + (xy)\,dy =$$

10. (15 pts.) Locate and classify the critical points of the function $f(x,y) = x^3 + y^3 - 3xy$. Use the second partials test in making your classification. (Fill in the table below after you locate all the critical points.)

Crit.Pt.	f_{xx} @ c.p.	f_{yy} @ c.p.	f_{xy} @ c.p.	Δ @ c.p.	Conclu-sion

11. (10 pts.) Let **F**$(x,y) = < 4\cdot\cos(x), 3\cdot\sin(y) >$.
(a) Show that the field is conservative by producing a potential function $\phi(x,y)$ so that $\nabla\phi(x,y) =$ **F**(x,y) for all (x,y) in the plane.

(b) Working within the influence of the force field from part (a), you move a particle along a curve C given by

$$\mathbf{r}(t) = < \pi\cdot\cos(t), 3\pi\cdot\sin(t) >$$

from time $t_0 = 0$ to time $t_1 = \pi/2$. How much work did you do?? [Hint: Where did you start? Where did you stop??]

12. (10 pts.) (a) Obtain parametric equations for the normal line to the surface defined by the equation $4x^2 + y^2 - 3z^2 = 2$ when the normal line passes through the point (-1,-1,-1) which is actually on the surface.

(b) The plane that is tangent at (-1,-1,-1) to the surface defined by $4x^2 + y^2 - 3z^2 = 2$ intersects the xy-plane in a straight line. Obtain a set of parametric equations for this line.

13. (10 pts.) Let $\mathbf{F}(x,y,z) = \langle 0 , e^{xy}\sin(y) , e^{xy}\cos(z) \rangle$.
Compute the divergence and the curl of the vector field **F**.

(a) div **F** =

(b) curl **F** =

14. (10 pts.) Compute the surface area of the part of the plane defined by $z = 10 - 2x - 5y$ that lies in the first octant.

15. (10 pts.) Use the substitution u = xy and v = y/x to compute the area of the region R in the first quadrant bounded by the lines defined by y = 2x and y = 4x, and the hyperbolas defined by xy = 1 and xy = 3. Thus, evaluate the following integral by doing the suggested substitution.

$$\iint_R 1 \, dx\, dy \;=$$

Silly 20 Point Bonus: Let

$$\mathbf{F}(x,y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$$

for $(x,y) \neq (0,0)$. You can easily verify that

$$\frac{\partial}{\partial y}\left[\frac{-y}{x^2+y^2}\right] = \frac{\partial}{\partial x}\left[\frac{x}{x^2+y^2}\right]$$

for $(x,y) \neq (0,0)$. You can also see that if $\phi(x,y) = \tan^{-1}(y/x)$, then for $x > 0$, $\nabla\phi(x,y) = \mathbf{F}(x,y)$. (a) Despite all this noise, prove that **F** is not a conservative field on the plane with the origin, (0,0), removed. (b) Also, prove that **F** is conservative on the upper half plane consisting of the set

$$U = \{ (x,y) \varepsilon\ \mathbb{R}^2 : y > 0 \}$$

by obtaining a potential for **F** on this set.
[Moral: Where you play matters.]