Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Vector objects must be denoted by using arrows. Since the answer really consists of all the magic transformations, do not "box" your final results. Show me all the magic on the page.

1. (10 pts.) (a) Obtain parametric equations for the line tangent to the graph of the helix defined by

$$\mathbf{r}(t) = < \cos(t), \sin(t), t/\pi >$$

when $t = \pi$.

(b) Does the tangent line at $t = \pi$ in part (a) intersect the plane defined by $x + y + z = 0$? If "yes", where??

2. (10 pts.) (a) Find the unit vectors $\mathbf{T}(t)$ and $\mathbf{N}(t)$ and the curvature $\kappa(t)$ for the helix defined by

$$\mathbf{r}(t) = < \cos(t), t, \sin(t)>.$$

(b) Locate the center $C(x_0,y_0,z_0)$ of the circle of curvature when $t = \pi/2$.

3. (10 pts.) (a) Obtain an equation for the plane containing the point (3,2,1) and parallel to the plane defined by the equation $10x - 25y + 13z = 75$.

(b) Obtain parametric equations for the line containing the point (3,2,1) and parallel to the line defined by the symmetric equations

$$\frac{x-7}{-4} = \frac{y+23}{-11} = \frac{z-\pi}{8\pi}$$

4. (10 pts.) (a) Obtain an equation for the plane that contains the three points (1,-1,1), (0,2,0), and (0,0,3).

(b) What is the area of the triangle in 3-space with vertices at (1,-1,1), (0,2,0), and (0,0,3). [Hint: You may already have done most of the work.]

5. (10 pts.) Obtain an arc-length parameterization for the curve **r**(t) = < t, cos(t), sin(t) > in terms of the initial point (π, -1, 0). Rather than overloading the symbol **r**, write this new parameterization as **R**(s). How are **R** and **r** related?

6. (10 pts.) (a) Find the center and radius of the sphere with the following equation:

$$x^2 + y^2 + z^2 + 4x - 6y + 8z = 0$$

(b) Obtain an equation for the sphere that has center at the point (1,-1,1) and that is tangent to the line in 3-space that is defined by the vector equation < x, y, z > = t< 1, 2, 3>.

7. (10 pts.) Write **c** in the form r**a** + s**b** where r and s are real numbers when **a** = <2,2>, **b** = <-2,2>, and **c** = <-3,2>. That is, write **c** as a linear combination of **a** and **b** if possible.

8. (10 pts.) (a) Find the direction angles of the vector represented by $\vec{PQ}$ when the points P(1,-1,0) and Q(3,4,5) are given.

(b) What point $R(x_0,y_0,z_0)$ is 2/3 of the way from P to Q??

9. (10 pts.) Suppose that an object is moving in a fixed plane with its acceleration given by **a**(t) = -32**j**. Suppose that the initial position of the object is **r**(0) = <1,0> and the initial velocity of the object is **v**(0) = < 2, 4>.

(a) Find the velocity of the object, **v**(t), as a function of time.

(b) Find the position of the object, **r**(t), as a function of time.

(c) Obtain an equation for the parabola that is the path of the object.

10. (10 pts.) (a) Find the exact value of the acute angle θ of intersection of the two planes defined by the two equations

$$x - 2y = -5 \quad \text{and} \quad y - 4z = 20.$$

θ =

(b) Obtain a set of parametric equations for the line of intersection of the two planes in part (a).

Silly 10 Point Bonus: Jeapardy!! The strange parameterization of the unit circle minus the point (-1,0) given by

$$\vec{r}(t) = <\frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2}> \ for\ -\infty < t < \infty$$

may be obtained using *two* quite different ideas. Reveal this magic in detail. Hint: Substitution deviousness & a different thought of lines do the jobs. Say where your work is here: