Name: Ogre Ogre TEST2/MAC2313 Bonus Business

Silly 10 Point Bonus: Suppose that f(x,y) = g(x), where g is the function defined on the real line by g(x) = 1 if $x \neq 0$ and g(0) = 0. (a) Using the definition, show

$$\lim_{x \to 0} g(x) = 1 .$$

(b) Then prove that if y_0 is any real number, f(x,y) does not have a limit as (x,y) ⟶> $(0,y_0)$. [This doesn't require use of the definition!] (c) Jeopardy: To which *natural* proposition does the pair of functions f and g provide a counter-example? Say where your work is here:

(a): Remember the definition?? Ouch. Proving that

$$\lim_{x \to 0} g(x) = 1$$

using the definition is truly easy. To this end, let $\varepsilon > 0$ be arbitrary. Let $\delta > 0$ be any positive number. We now show that this δ has the desired property. If x is any real number with $0 < |x - 0| < \delta$, then from the definition of the function g above, $|g(x) - 1| = |1 - 1| = 0 < \varepsilon$. //

(b): To see f(x,y) does not have a limit as (x,y) ⟶> $(0,y_0)$, it suffices to compute the limits along the two lines x = 0 and $y = y_0$ as (x,y) ⟶> $(0,y_0)$ and observe that these limits exist and have different values. Evidently,

$$\lim_{\substack{(x,y) \to (0,y_0) \\ x = 0}} f(x,y) = \lim_{y \to y_0} f(0,y)$$

$$= \lim_{y \to y_0} g(0) = 0,$$

and

$$\lim_{\substack{(x,y) \to (0,y_0) \\ y = y_0}} f(x,y) = \lim_{x \to 0} f(x,y_0)$$

$$= \lim_{x \to 0} g(x) = 1.$$

Of course, if you want to, you *can* use the definition to prove the limit doesn't exist.

(c): Jeopardy! **Proposition:** If f(x,y) = g(x), where g is defined in an interval containing $x = x_0$, and

$$\lim_{x \to x_0} g(x) = L,$$

then $f(x,y) \to L$ as $(x,y) \to (x_0,y_0)$.

Clearly, parts (a) and (b) provide a counter-example for this, and it appears to be a *natural* proposition that one might, at least initially, suppose to be true.