Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations. Write using complete sentences. Remember this: "=" denotes "equals" , "$\Rightarrow$" denotes "implies" , and "$\Leftrightarrow$" denotes "is equivalent to". Vector objects must be denoted by using arrows. Since the answer really consists of all the magic transformations, do not "box" your final results. Show me all the magic on the page.

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1. (10 pts.) (a) If the cylindrical coordinates for a point P are given by $(r, \theta, z) = (3, (7/6)\pi, -1)$, then the rectangular coordinates of the point are given by

$(x, y, z) =$

(b) If the spherical coordinates of a point for a point P are given by $(\rho, \phi, \theta) = (4, (1/6)\pi, (2/3)\pi)$, then the rectangular coordinates of the point are given by

$(x, y, z) =$

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2. (10 pts.) (a) Suppose the rectangular coordinates for a point P are given by $(x, y, z) = (-2, -1, 2)$. Obtain cylindrical and spherical coordinates for this point. In doing this, ensure that $0 \leq \theta < 2\pi$.

(b) Describe the graph of the equation $\rho = 4\cos(\phi)$ as precisely as possible. [Evidently this is in spherical coordinates.] You may wish to convert this equation to an equivalent rectangular equation to do this.

3. (10 pts.) (a) Write an equation for the surface generated when the curve defined by the equation $y = 4 - x^2$ in the x,y - plane is revolved around the y - axis.

(b) Do the three 2-space sketches of the traces in each of the coordinate planes of the surface defined by $z = 1 - x^2 - y^2$. Work below and label carefully. Then on the back of page 1 attempt to do a 3 - space sketch in the plane of the surface.

4. (10 pts.) (a) What is the largest possible domain of the function $f(x,y) = \ln(x^2 + y^2 - 4)$

(2) Consider the function $f(x,y) = x^2 - y$. Obtain an equation for the level curve for this function through the point (1,-2) in the x,y - plane. [Hint: What is the *level* for the level curve?]

5. (10 pts.) (a) Using complete sentences and appropriate notation, give the ε - δ definition for

(*) $$\lim_{(x,y) \to (a,b)} f(x,y) = L.$$

(b) In the example where the authors were showing that

$$\lim_{(x,y) \to (a,b)} xy = ab,$$

they asserted that if they took f(x,y) = x and g(x,y) = y, then it followed from the definition of limit that

(i) $\lim_{(x,y) \to (a,b)} f(x,y) = a$ and (ii) $\lim_{(x,y) \to (a,b)} g(x,y) = b.$

Choose one of equations (i) or (ii), indicate to me which you have chosen, and then prove the equation is true using the definition.

6. (10 pts.) (a) If f(x,y) is a function of two variables, state the definition of the partial derivative of f with respect to y.

(b) Let $f(x,y) = xy^2 - 2$. Using the definition, compute $f_y(x,y)$.

7. (10 pts.) Find an equation for the plane tangent to the surface $z = f(x,y)$ at the point $P = (5, 2, 9)$ when

$$f(x,y) = x^2 - 4y^2.$$

8. (10 pts.) (a) Evaluate the following limit.

$$\lim_{(x,y)\to(-1,2)} \frac{5 - x^2}{1 + xy} =$$

(b) Is f defined by $f(x,y) = \begin{cases} \dfrac{\tan(3(x^2+y^2))}{x^2+y^2} & , (x,y)\neq(0,0) \\ 2 & , (x,y)=(0,0) \end{cases}$

continuous at (0,0)? A complete explanation is required. Details ... .

9. (10 pts.) Find every point on the surface z = f(x,y) where the tangent plane is horizontal when

$$f(x,y) = 3x^2 + 12x + 4y^3 - 6y^2.$$

10. (10 pts.) Reveal how one finds the maximum and minimum values of f(x,y) = 2xy on the circular disk R, where $R = \{ (x,y) : x^2 + y^2 \le 4 \}$. Oh yes, do find the maximum and minimum values, too. [Hint: Is an arctic boundary cool?]

Silly 10 Point Bonus: Suppose that f(x,y) = g(x), where g is the function defined on the real line by g(x) = 1 if $x \neq 0$ and g(0) = 0. (a) Using the definition, show

$$\lim_{x \to 0} g(x) = 1 \ .$$

(b) Then prove that if y_0 is any real number, f(x,y) does not have a limit as $(x,y) \longrightarrow (0,y_0)$. [This doesn't require use of the definition!] (c) Jeopardy: To which *natural* proposition does the pair of functions f and g provide a counter-example? Say where your work is here: