Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Generic vector objects must be denoted by using arrows. Since the answer really consists of all the magic transformations, do not "box" your final results. Show me all the magic on the page neatly.

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1. (10 pts.)
Let $f(x,y) = (x^2 + y^2)^{1/2}$. Compute the gradient of f at (4,-3), and then use it to compute $D_{\mathbf{u}}f(4,-3)$, where **u** is the unit vector in the same direction as **v** = <-12,5>.

$\nabla f(x,y) =$

$\nabla f(3,-4) =$

u =

$D_{\mathbf{u}}f(3,-4) =$

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2. (10 pts.)
Let $f(x,y) = e^{-3x + 4y}$ and let $P_0 = (0,0)$.
(a) Compute the rate of change of f(x,y) at P_0 in the direction in which f(x,y) increases most rapidly.

(b) Find a unit vector in the direction in which f(x,y) increases most rapidly at P_0.

3. (10 pts.) Make the gradient do a perp walk.

(a) When $f(x,y) = x^4 + xy + y^2$, compute $\nabla f(2,-3)$. Label your expressions correctly. Don't scratch.

(b) Use the result of (a) to produce an equation for the line tangent to the graph of $x^4 + xy + y^2 = 19$ at the point $(x_0,y_0) = (2,-3)$.

(c) Use the result of (a) to produce an equation for the line perpendicular to the graph of $x^4 + xy + y^2 = 19$ and passing through the point $(x_0,y_0) = (2,-3)$. [Hint: There is no restriction on the *kind* of equation.]

4. (10 pts.) (a) Find the differential, dw, of

$$w = (x^{1/2} + y^{1/2})^2.$$

(b) Reveal, in detail, how to use differentials to approximate

$$((17)^{1/2} + (24)^{1/2})^2.$$

5. (10 pts.) Locate and classify the critical points of the function $f(x,y) = x^3 + 3xy^2 - 3x$. Use the second partials test in making your classification. (Fill in the table below after you locate all the critical points.

Crit.Pt.	f_{xx} @ c.p.	f_{yy} @ c.p.	f_{xy} @ c.p.	Δ @ c.p.	Conclusion

6. (10 pts.) (a) Compute $\partial z/\partial x$ and $\partial z/\partial y$ as functions of x, y, and z assuming that $z = f(x,y)$ when $x^2 + y^2 + z^2 = 9$. [These are implicit differentiations, of course.]

(b) Using your results from part (a), obtain an equation for the plane tangent to the surface defined by $x^2 + y^2 + z^2 = 9$ at the point $P(1,2,2)$.

7. (10 pts.) By using Lagrange multipliers, find the extreme values of $f(x,y) = y^2 - x^2$ and precisely where they occur on the circle defined by the equation $x^2 + y^2 = 4$.

8. (10 pts.) Chain, chain, chain, chain of ... .

(a) Suppose that $w = \ln(2x + 3y)$, $x = r\cdot\cos(\theta)$ and $y = r\cdot\sin(\theta)$. Compute $\partial w/\partial\theta$.

$\partial w/\partial\theta$ =

(b) Suppose that $x = h(y,z)$ satisfies the equation $F(x,y,z) = 0$, and that $F_x \neq 0$. Show how to compute $\partial x/\partial z$ in terms of the partial derivatives of F.

9. (10 pts.) It turns out that $h(y) = \sin(y)/y$ does not have an elementary antiderivative. Despite that you can evaluate the following iterated integral. Do this by reversing the order of integration and then evaluating the new iterated integral you obtain. [Hint: It helps to sketch the region of integration, R.]

$$\int_0^{\pi}\int_x^{\pi} \frac{\sin(y)}{y}\, dy\, dx =$$

10. (10 pts.) (a) Let R be the region bounded by the curves defined by $y = x^2$ and $y = x + 2$. Write an iterated double integral that gives the area of the region, but do not attempt to evaluate the iterated integral.

$$area(R) = \iint_R 1\, dA =$$

(b) Set up, but do not attempt to evaluate the iterated double integral that will give the numerical value for the volume of the solid bounded by the cylinders in 3-space defined by $x^2 + y^2 = 1$ and $y^2 + z^2 = 1$. [Hint: You should only need to sketch the xy-cylindrical stuff to obtain the limits of integration.]

V =

Silly 10 Point Bonus: Suppose $f(x,y)$ is differentiable at an interior point (x_0,y_0) in its domain. Pretend there are at least three distinct unit vectors **u** satisfying the following equation: $D_{\mathbf{u}}f(x_0,y_0) = 0$. Does it follow as a consequence that this equation must be true for all unit vectors? Proof?? Where???