
Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Generic vector objects must be denoted by using arrows. Since the answer really consists of all the magic transformations, do not "box" your final results. Show me all the magic on the page neatly.

Silly 10 Point Bonus: What's the numerical value of the following double integral, where R is the first quadrant.

$$\iint_R e^{-(x^2+y^2)}\,dA$$

[Where? You don't have room here to reveal the needed details.]

Of course if you read with care Example 5 of Section 13.4 (Section 14.4 of the paperback beast), you would find this a snooze. Evidently, the integral is improper. By converting to polar coordinates, we have

$$\begin{aligned}
\iint_R e^{-(x^2+y^2)}\,dA &= \int_0^{\pi/2}\int_0^{\infty} e^{-r^2} r\,dr\,d\theta \\
&= \lim_{b\to\infty} \int_0^{\pi/2}\int_0^{b} e^{-r^2} r\,dr\,d\theta \\
&= \lim_{b\to\infty} \int_0^{b} e^{-r^2} r\,dr \cdot \int_0^{\pi/2} 1\,d\theta \\
&= \lim_{b\to\infty} \left(\frac{\pi}{2}\right)\left(\frac{1}{2} - \frac{e^{-b^2}}{2}\right) \\
&= \frac{\pi}{4}.
\end{aligned}$$

Of course there are some technical thingies that we are ignoring, but that's okay here.