Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations. Write using complete sentences. Remember this: "=" denotes "equals" , "$\Rightarrow$" denotes "implies" , and "$\Leftrightarrow$" denotes "is equivalent to". Vector objects must be denoted by using arrows. Since a correct answer really consists of all the magic transformations, do not "box" your final results. Show me all the magic on the page clearly.

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1. (10 pts.) Obtain an equation for the plane that is tangent to the sphere with equation

$$(x - 1)^2 + (y - 2)^2 + (z - 3)^2 = 51$$

at the point (2, 3, -4). [Hint: Who’s normal?]

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2. (10 pts.) Obtain parametric equations for the line obtained when the two planes defined by $x + 3y + 6z = 10$ and $x - 2y + z = 5$ intersect.

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3.   (10 pts.)   (a) Obtain parametric equations for the line that contains the point (25, -24, 23) and the center of the sphere defined by the following equation:

$$x^2 + y^2 + z^2 + 4x - 6y + 8z = 0$$

(b) What is the radius of the sphere that has its center at the point (1,2,3) and that is tangent to the line in 3-space that is defined by the vector equation $< x, y, z > = t< 1, -1, 1>$.

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4.   (10 pts.)   Suppose that an object is moving in a fixed plane with its acceleration given by $\mathbf{a}(t) = <0,-32>$. Suppose that the initial position of the object is $\mathbf{r}(0) = <0,1>$ and the initial velocity of the object is $\mathbf{v}(0) = < 2, 2>$.

(a) Find the velocity of the object, $\mathbf{v}(t)$, as a function of time.

(b) Find the position of the object, $\mathbf{r}(t)$, as a function of time.

(c) Obtain an equation for the parabola that is the path of the object.

5. (10 pts.) Obtain an arc-length parameterization for the curve **r**(t) = < cos(t), 2t, sin(t) > in terms of the initial point (1, 4π, 0). Rather than overloading the symbol **r**, write this new parameterization as **R**(s). How are **R** and **r** related? [Hint: What is the parameter t_0 that gives the initial point in vector form, i.e., **r**(t_0) = < 1, 4π, 0> ?? It's not zero, Folks.]

6. (10 pts.) Let **r**(t) = <4·cos(t) , 4·sin(t), 3·t> for $t \varepsilon \mathbb{R}$. Then

(a) **r**′(t) = ,

(b) **r**″(t) = ,

(c) **T**(t) = ,

(d) **N**(t) = , and

(e) κ(t) = .

7. (10 pts.) A particle moves smoothly in such a way that at a particular time t = 0, we have **r**(0) = < 0 , 1>, **v**(0) = < 1 , 0> and **a**(0) = < 0 , -2>. If we write **a**(0) in terms of **T**(0) and **N**(0), then

$$\mathbf{a}(0) = a_T(0)\mathbf{T}(0) + a_N(0)\mathbf{N}(0),$$

where

(a) **T**(0) = ,

(b) $a_T(0)$ = ,

(c) $a_N(0)$ = , and

(d) **N**(0) = .

(e) Consequently, an equation for the circle of curvature is given by:

8. (10 pts.) (a) Describe the graph of the equation

$$r = 4\cos(\theta) - 6\sin(\theta)$$

as precisely as possible. [We are playing in the cylindrical world, not merely on a polar ice floe.] You may wish to convert this equation to an equivalent rectangular equation to do this.

(b) What are the precise spherical coordinates of the point in 3-space with rectangular coordinates (x, y, z) = (-1, -2, -3)? In doing this, ensure that $0 \le \theta < 2\pi$.
[Hint: You will have to express your answer using your friends, $\cos^{-1}$ and $\tan^{-1}$. The angles are rude today and not given to pious niceties.]

9. (10 pts.) Write an equation for the surface generated when the curve defined by the equation $x = 4 - y^2$ in the x,y - plane is revolved around the x - axis.

10. (10 pts.) Do the three 2-space sketches of the traces in each of the coordinate planes of the surface defined by $z = x^2 + y^2 - 1$. Work below and label carefully. Then on the back of page 4 attempt to do a 3 - space sketch in the plane of the surface.

Silly 10 Point Bonus: Your friendly garden variety parabola, $y = x^2$, is given in vector form by the equation

$$\boldsymbol{r}(t) = \langle t, t^2 \rangle,\ t \in \mathbb{R}.$$

Perform the algebraic magic of obtaining the center of the osculating circle as a function of *t*. [Hint: You need the principal normal as a function of *t* and curvature, too.] Where's your work? It won't fit here!!