
Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Vector objects must be denoted by using arrows. Do not "box" your final results. Show me all the magic on the page.

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1. (10 pts.) (a) What is the largest possible domain of the function $f(x,y) = (9 - x^2 - y^2)^{1/2}$?

(b) Consider the function $f(x,y) = x - y^2$. Obtain an equation for the level curve for this function that passes through the point (1,-2) in the x,y - plane. [Hint: What is the *level* for the level curve?]

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2. (10 pts.)
Let $f(x,y) = (x^2 + y^2)^{1/2}$. Compute the gradient of f at (-12,5), and then use it to compute $D_{\mathbf{u}}f(-12,5)$, where **u** is the unit vector in the same direction as **v** = <-4,3>.

$\nabla f(x,y) =$

$\nabla f(-12,5) =$

u =

$D_{\mathbf{u}}f(-12,5) =$

3. (10 pts.) (a) If $f(x,y)$ is a function of two variables, state the definition of the partial derivative of f with respect to x.

(b) Let $f(x,y) = x^2y^2 - 2y$. Using only the definition, reveal all the details of the computation of $f_x(x,y)$.

4. (10 pts.)
Let $f(x,y) = \tan^{-1}(2x+3y)$. (a) Compute the total differential, df, of the function f.

$df =$

(b) Use differentials to approximate the numerical value of $f(.1,-.2)$ when f is the function of part (a) of this problem.

5. (10 pts.) (a) Using complete sentences and appropriate notation, give the ε - δ definition for

(*) $$\lim_{(x,y)\to(a,b)} f(x,y) = L.$$

(b) In the example where Edwards and Penney were showing that

$$\lim_{(x,y)\to(a,b)} xy = ab$$

they asserted that if they took $f(x,y) = x$ and $g(x,y) = y$, then it followed from the definition of limit that

(i) $\lim_{(x,y)\to(a,b)} f(x,y) = a$ and (ii) $\lim_{(x,y)\to(a,b)} g(x,y) = b$.

Choose one of equations (i) or (ii), indicate to me which you have chosen, and then prove the equation is true using the ε-δ definition.

6. (10 pts.) Let $f(x,y) = \sin(3x + 4y)$ and let $P_0 = (0 , 0)$.
(a) Find a unit vector in the direction in which f(x,y) decreases most rapidly at P_0.

(b) Compute the rate of change of f(x,y) at P_0 in the direction in which f(x,y) increases most rapidly.

7. (10 pts.) Obtain an equation for the tangent plane to the surface defined by the equation $5x^2 + 6y^2 - 2z^2 = 9$ at the point $(-1, 1, -1)$ which is on the surface.

8. (10 pts.) Locate and classify the critical points of the function $f(x,y) = 2x^4 + y^2 - 4xy$. Use the second partials test in making your classification. (Fill in the table below after you locate all the critical points.)

Crit.Pt.	f_{xx} @ c.p.	f_{yy} @ c.p.	f_{xy} @ c.p.	Δ @ c.p.	Conclu-sion

9. (10 pts.) (a) Suppose that $y = h(x,z)$ satisfies the equation $F(x,y,z) = 0$, and that $F_y \neq 0$. Show how to compute $\partial y/\partial z$ in terms of the partial derivatives of F.

(b) Is f defined by $f(x,y) = \begin{cases} \dfrac{\sin(5(x^2+y^2))}{x^2+y^2} & , (x,y)\neq(0,0) \\ 5 & , (x,y)=(0,0) \end{cases}$

continuous at (0,0)? A complete explanation is required. Details & definitions are at the heart of it.

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10. (10 pts.) Obtain and locate the absolute extrema of the function $f(x,y) = (x - y)^2$ on the closed disk D with radius five centered at the origin. Observe that D is given in set builder notation as follows: $D = \{ (x,y) : x^2 + y^2 \leq 25 \}$. In performing this magic, use Lagrange multipliers to deal with the behavior of f on the boundary, $B = \{ (x,y) : x^2 + y^2 = 25 \}$.

Silly 10 Point Bonus: Suppose that $(x_0,y_0) \neq (0,0)$. Without using any of the usual tools of Calculus, like the derivative, obtain the absolute extrema of the function

$$f(\theta) = x_0\cos(\theta) + y_0\sin(\theta)$$

and locate the precise θ's in the interval $[0,2\pi)$ where the extrema occur. [Indicate where your work is.]