Name:

STUDENT #: EXAM #:

Read Me First: Use correct notation when presenting your computations. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Vector objects must be denoted by using arrows. Since the answer really consists of all the magic incantations and transformations, do not "box" your final results. Show me all the magic on the page neatly.

1. (15 pts.) Evaluate the following line integral, where C is the path from (1,-1) to (1,1) along the curve $x = y^2$. [Hint: The field is not conservative. The path is not closed. Parameterize C first. A picture might help -- or not.]

$$\int_C (2xy)\,dx + (x - y)\,dy =$$

2. (15 pts.) Convert the double integral

$$\int_{-1}^{1}\int_{0}^{(1-x^2)^{1/2}} \cos\left(\frac{\pi}{2}(x^2 + y^2)\right)\,dy\,dx =$$

to an equivalent integral in polar coordinates, and then evaluate the integral you obtain. A picture of the region helps.

$$\int_{-1}^{1}\int_{0}^{(1-x^2)^{1/2}} \cos\left(\frac{\pi}{2}(x^2 + y^2)\right)\,dy\,dx =$$

3. (15 pts.) A particle, starting at (0,0) moves to the point (0,1) along the y-axis, goes to the point (-1,0) by traveling along the circle $x^2 + y^2 = 1$, and then returns to (0,0) along the x-axis. Use Green's Theorem to compute the work done on the particle by the force field $\mathbf{F}(x,y) = \langle -y^3, x^3 \rangle$.

4. (15 pts.) Obtain an arc-length parameterization for the curve $\mathbf{r}(t) = \langle 4\cos(t), 4\sin(t), 3t \rangle$ in terms of the initial point $(-4, 0, 3\pi)$. Rather than overloading the symbol **r**, write this new parameterization as $\mathbf{R}(s)$. If you move along the curve using the parameterization given by **R**, what's your speed?

5. (15 pts.)

Let $f(x,y) = 3y^2 + 4x^2$.

(a) Compute the gradient of f at (1,-1). (b) Then use it to compute $D_{\mathbf{u}}f(1,-1)$, where $\mathbf{u} = \langle \cos(5\pi/6), \sin(5\pi/6) \rangle$ is the unit vector that forms an angle of $5\pi/6$ with respect to the positive x-axis. (c) Finally, obtain a unit vector $\mathbf{v}_0$ so that $D_{\mathbf{v}}f(1,-1)$ is a minimum as $\mathbf{v}$ is allowed to range over all unit vectors in the plane.

$\nabla f(x,y)$ =

$\nabla f(1,-1)$ =

$\mathbf{u}$ =

$D_{\mathbf{u}}f(1,-1)$ =

$\mathbf{v}_0$ =

6. (15 pts.) Write down but do not attempt to evaluate the iterated triple integrals in (a) rectangular, (b) cylindrical, and (c) spherical coordinates that would be used to compute the volume enclosed between the xy-plane and the upper half of the sphere defined by $x^2 + y^2 + z^2 = (10)^2$ in the first octant. This means the solid is bounded by the three coordinate planes as well as the sphere. [*Upper* means $z \geq 0$.]

(a) V =

(b) V =

(c) V =

7. (15 pts.) Determine the maximum and minimum values of $f(x,y) = 2xy$ and where they are obtained when (x,y) lies in the closed disk defined by $x^2 + y^2 \le 1$. Analyze f on the boundary by using Lagrange Multipliers.

8. (15 pts.) (a) (12 pts.) Compute the unit vectors **T**(t) and **N**(t) and the curvature $\kappa(t)$ for the helix defined by

$$\mathbf{r}(t) = \langle 12 \cdot \cos(t), 5t, 12 \cdot \sin(t) \rangle.$$

(b) (3 pts.) Locate the center $C(x_0,y_0,z_0)$ of the circle of curvature when $t = \pi$.

9. (15 pts.) Obtain a set of parametric equations for the line through the point (0, 0, 0) that is perpendicular to the plane that contains the three points (1,0,0), (0,2,0), and (0,3,0). Then locate the point in 3-space where the line intersects the plane.

10. (15 pts.) Locate and classify the critical points of the function $f(x,y) = xy^2 - y^2 - 8x^2$. Use the second partials test in making your classification. (Fill in the table below after you locate all the critical points. It always helps to factor. How does a product equal zero??)

Crit.Pt.	f_{xx} @ cp	f_{yy} @ cp	f_{xy} @ cp	Δ @ cp	Conclu-sion

11. (10 pts.) Let **F**(x,y) = < 6x + 1, 4y + 1 >.
(a) Show that the field is conservative by producing a potential function $\phi(x,y)$ so that $\nabla\phi(x,y) = \mathbf{F}(x,y)$ for all (x,y) in the plane.

(b) Working within the influence of the force field from part (a), you move a particle along an ellipse C given by

$$\mathbf{r}(t) = < (3\cos(t), 4\sin(t) >$$

from time $t_0 = \pi/2$ to time $t_1 = \pi$. How much work did you do??

12. (10 pts.)
Let **F**(x,y,z) = < 0, $e^{xy}\cos(z)$, $e^{xy}\sin(z)$ >. Compute the divergence and the curl of the vector field **F**.

(a) div **F** =

(b) curl **F** =

13. (10 pts.) (a) Obtain an equation for the plane that is tangent to the surface defined by the equation $4x^2 + y^2 - 3z^2 = 2$ at the point (1, 1, -1).

(b) Does the plane that is tangent at (1, 1, -1) to the surface defined by $4x^2 + y^2 - 3z^2 = 2$ intersect the yz-plane? If the answer is "yes", obtain a set of parametric equations in 3-space for the line of intersection.

14. (10 pts.) Compute the following surface integral, where S is the portion of the plane x + y + z = 1 lying over the first quadrant.

$$\iint_S z \, dS =$$

15. (10 pts.) Let R be the triangular region enclosed by the lines $y = 0$, $y = x$, and $x + y = 1$. Use a suitable transformation to compute the following integral:

$$\iint_R \exp\left(\frac{x-y}{x+y}\right)\, dA =$$

Silly 20 Point Bonus: Let

$$\mathbf{F}(x,y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$$

for $(x,y) \neq (0,0)$. You can easily verify that

$$\frac{\partial}{\partial y}\left[\frac{-y}{x^2+y^2}\right] = \frac{\partial}{\partial x}\left[\frac{x}{x^2+y^2}\right]$$

for $(x,y) \neq (0,0)$. You can also see that if $\phi(x,y) = \tan^{-1}(y/x)$, then for $x > 0$, $\nabla\phi(x,y) = \mathbf{F}(x,y)$. (a) Despite all this noise, prove that **F** is not a conservative field on the plane with the origin, (0,0), removed. (b) Also, prove that **F** is conservative on the upper half plane consisting of the set

$$U = \{ (x,y) \varepsilon\ \mathbb{R}^2 : y > 0 \}$$

by obtaining a potential for **F** on this set. (c) It turns out that as long as you restrict (x,y) to any disk D that does not contain the point (0,0) the field **F** has a potential function $\phi(x,y)$. Give an explicit formula for ϕ when this is the case.
[Moral: Where you play matters.] Say Where your work is, for it won't fit here.