Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". *Vector objects must be denoted by using arrows.* Since a correct answer really consists of all the magic transformations, do not "box" your final results. Show me all the magic on the page clearly.

Silly 10 Point Bonus: Plainly,

$$\begin{cases} x = 3 + \cos(t) \\ y = -4 + \sin(t)\ , \quad t \in [0, 2\pi] \end{cases}$$

and

$$\begin{cases} \arccos(x-3) = t \\ \qquad y = -4 + \sin(\arccos(x-3)) \end{cases}$$

are not equivalent. How must the second parameterization be altered so as to yield the same curve as that defined by the first parameterization?? [Say where your work is, for it won't fit here.]

The key observation regarding the second parameterization is that the range of $\cos^{-1}$ restricts t to the interval $[0,\pi]$ so that only the top half of the circle defined by

$$(x - 3)^2 + (y + 4)^2 = 1$$

gets traced counter-clockwise as t runs from 0 to π. What the second parameterization needs is the following missing piece to continue around the circle as t goes from π to 2π:

$$\begin{cases} \arccos(x-3) = 2\pi - t \\ \qquad y = -4 + \sin(2\pi - \arccos(x-3)) \ \textit{for}\ t \in [\pi, 2\pi]. \end{cases}$$

Where did the "2π - t" come from?? Play with the pieces!! Check $t = \pi$, $t = 3\pi/2$, and $t = 2\pi$. Eliminate the parameter t from each "piece". Think. Here is is the second parameterization properly done:

$$\begin{cases} \arccos(x-3) = t \\ \qquad y = -4 + \sin(\arccos(x-3)) \quad \textit{for}\ t \in [0, \pi] \\ \\ \arccos(x-3) = 2\pi - t \\ \qquad y = -4 + \sin(2\pi - \arccos(x-3)) \ \textit{for}\ t \in (\pi, 2\pi]. \end{cases}$$

Of course there are correct equivalent alternatives to this.