Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". *Vector objects must be denoted by using arrows.* Since a correct answer really consists of all the magic transformations, do not "box" your final results. Show me all the magic on the page clearly.

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1. (10 pts.) Suppose that an object is moving in a fixed plane with its acceleration given by **a**(t) = <0,-32>. Suppose that the initial position of the object is **r**(0) = <0,1> and the initial velocity of the object is **v**(0) = < 2, 2>.

(a) (6 pts.) Find the velocity of the object, **v**(t), as a function of time.

(b) (2 pts.) Find the position of the object, **r**(t), as a function of time.

(c) (2 pts.) Obtain an equation for the parabola that is the path of the object.

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2. (10 pts.) Obtain an equation for the plane that contains the points (1,0,0), (1,-2,0), and (1,-2,3).

3. (10 pts.) Let $\mathbf{r}(t) = < 3\cos(t) , 3\sin(t), 4t >$ for $t \varepsilon \mathbb{R}$. Then

(a) $\mathbf{r}'(t) =$,

(b) $\mathbf{r}''(t) =$,

(c) $\mathbf{T}(t) =$,

(d) $\mathbf{N}(t) =$, and

(e) $\kappa(t) =$.

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4. (10 pts.) (a) Determine the radius and the center of the sphere defined by the following equation:

$$x^2 + y^2 + z^2 + 8x - 6y + 10z = 0$$

(b) What is the parameter t_0 of the point on the line defined by the vector equation $< x, y, z > = t< 1, 1, 1>$ that is nearest to the point (1,3,2)?

5. (10 pts.) Obtain an arc-length parameterization for the curve $\mathbf{r}(t) = < 3t, \cos(t), \sin(t) >$ in terms of the initial point (0, 1, 0). Rather than overloading the symbol **r**, write this new parameterization as $\mathbf{R}(s)$. How are **R** and **r** related?

6. (10 pts.) Obtain parametric equations for the line of intersection of the two planes defined by $x + 3y - 6z = 10$ and $y + 2z = 5$.

7. (10 pts.) A particle moves smoothly in such a way that at a particular time t = 0, we have **r**(0) = < 1 , 0>, **v**(0) = < 1 , 1> and **a**(0) = < 2 , -1>. If we write **a**(0) in terms of **T**(0) and **N**(0), then

$$\mathbf{a}(0) = a_T(0)\mathbf{T}(0) + a_N(0)\mathbf{N}(0),$$

where

(a) **T**(0) = ,

(b) $a_T(0)$ = ,

(c) $a_N(0)$ = , and

(d) **N**(0) = .

(e) Consequently, an equation for the circle of curvature is given by:

8. (10 pts.) (a) Describe the graph of the equation

$$r = 10\cos(\theta) + 6\sin(\theta)$$

as precisely as possible. [We are playing in the cylindrical world, not merely on a polar ice floe.] You may wish to convert this equation to an equivalent rectangular equation to do this.

(b) What are the precise spherical coordinates of the point in 3-space with rectangular coordinates (x, y, z) = (-1, -2, -1)? In doing this, ensure that $0 \le \theta < 2\pi$.
[Hint: You will have to express your answer using your friends, $\cos^{-1}$ and $\tan^{-1}$. The angles are rude today and not given to pious niceties.]

9. (10 pts.) (a) Find the direction angles of the vector ***v*** = < -1, 3, -2 >.

(b) What is the angle θ between the two vectors ***v***, from part (a) above, and ***w*** = <1,2,1> ?

10. (10 pts.) Do the three 2-space sketches of the traces in each of the coordinate planes of the surface defined by $z = 1 - x^2 - y^2$. Work below and label very carefully. Then on the back of Page 4 attempt to do a 3 - space sketch in the plane of the surface.

Silly 10 Point Bonus: Plainly,

$$\begin{cases} x = 3 + \cos(t) \\ y = -4 + \sin(t), \quad t \in [0, 2\pi] \end{cases}$$

and

$$\begin{cases} \arccos(x-3) = t \\ y = -4 + \sin(\arccos(x-3)) \end{cases}$$

are not equivalent. How must the second parameterization be altered so as to yield the same curve as that defined by the first parameterization?? [Say where your work is, for it won't fit here.]