Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations. Write using complete sentences. Remember this: "=" denotes "equals" , "$\Rightarrow$" denotes "implies" , and "$\Leftrightarrow$" denotes "is equivalent to". Vector objects must be denoted by using arrows. Do not "box" your final results. Show me all the magic on the page.

1. (10 pts.) (a) If f(x,y) is a function of two variables, using a complete sentence and appropriate notation, state the definition of the partial derivative of f with respect to y.

(b) Let $f(x,y) = x^2y^2 - 2y$. Using only the definition, reveal all the details of the computation of $f_y(x,y)$.

2. (10 pts.) Let $f(x,y) = \sin(2x+3y)$. (a) Compute the total differential, df, of the function f.

$df =$

(b) Use a linear approximation to approximate the numerical value of f(.1,-.1) when f is the function of part (a) of this problem.

3. (10 pts.) (a) Suppose that $x = h(y,z)$ satisfies the equation $F(x,y,z) = 0$, and that $F_x \neq 0$. Show how to compute $\partial x/\partial y$ in terms of the partial derivatives of F.

(b) Is f defined by $f(x,y) = \begin{cases} \dfrac{\tan(\pi(x^2+y^2))}{2(x^2+y^2)} & ,\ (x,y)\neq(0,0) \\ \pi & ,\ (x,y)=(0,0) \end{cases}$

continuous at (0,0)? A complete explanation is required. Details & definitions are at the heart of it.

4. (10 pts.) Consider the function f of three variables defined by $f(x,y,z) = 5x^2 + 6y^2 - 4z^2$. Obtain an equation for the level surface for this function that passes through the point (1,-1, -1) in the 3 - space.

(b) Obtain an equation for the plane that is tangent to the level surface given in part (a) with the point of tangency being (1,-1, -1).

5. (10 pts.) (a) Using complete sentences and appropriate notation, give the ε - δ definition for

(*) $$\lim_{(x,y)\to(a,b)} f(x,y) = L.$$

(b) In the example where Edwards and Penney were showing that

$$\lim_{(x,y)\to(a,b)} xy = ab$$

they asserted that if they took $f(x,y) = x$ and $g(x,y) = y$, then it followed from the definition of limit that

(i) $\displaystyle\lim_{(x,y)\to(a,b)} f(x,y) = a$ and (ii) $\displaystyle\lim_{(x,y)\to(a,b)} g(x,y) = b$.

Choose one of equations (i) or (ii), indicate to me which you have chosen, and then prove the equation is true using the ε-δ definition.

6. (10 pts.) Let $f(x,y) = \tan(3x + 4y)$ and let $P_0 = (0,0)$.
(a) Find a unit vector in the direction in which $f(x,y)$ increases most rapidly at P_0.

(b) Compute the rate of change of $f(x,y)$ at P_0 in the direction in which $f(x,y)$ decreases most rapidly.

7. (10 pts.) Obtain and locate the absolute extrema of the function $f(x,y) = x - y$ on the closed disk D with radius five centered at the origin. Observe that D is given in set builder notation as follows: $D = \{ (x,y) : x^2 + y^2 \le 25 \}$. In performing this magic, use Lagrange multipliers to deal with the behavior of f on the boundary, $B = \{ (x,y) : x^2 + y^2 = 25 \}$.

8. (10 pts.) Locate and classify the critical points of the function $f(x,y) = 8xy - 2x^2 - y^4$. Use the second partials test in making your classification. [Fill in the table below after you locate all the critical points.]

Crit.Pt.	f_{xx} @ c.p.	f_{yy} @ c.p.	f_{xy} @ c.p.	Δ @ c.p.	Conclu-sion

9. (10 pts.) Let $f(x,y) = \sin(x)\cos(y)$. Compute the gradient of f at $(\pi/3, -2\pi/3)$, and then use it to compute $D_{\mathbf{u}}f(\pi/3, -2\pi/3)$, where $\mathbf{u}$ is the unit vector in the same direction as $\mathbf{v} = \langle 4,-3\rangle$.

$\nabla f(x,y) =$

$\nabla f(\pi/3, -2\pi/3) =$

$\mathbf{u} =$

$D_{\mathbf{u}}f(\pi/3, -2\pi/3) =$

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10. (10 pts.) Recall that if $f(x,y)$ has continuous second order partial derivatives, then

$$\frac{\partial^2 f}{\partial x\,\partial y}(x,y) = \frac{\partial^2 f}{\partial y\,\partial x}(x,y).$$

Does there exist such a function f such that $f_x(x,y) = 2xy^3$ and $f_y(x,y) = 3x^2y^2 + 1$?? If the answer is "yes", obtain a formula for f.

Silly 10 Point Bonus: Do at most one of the following problems. Indicate which unambiguously and say where your work is, for it won't fit here!

(a) Let g be defined by

$$g(x,y) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0). \end{cases}$$

Explicitly compute $g_{xy}(0,0)$ and $g_{yx}(0,0)$.

(b) Determine the absolute extrema of the function $f(x,y) = -y$ subject to the points (x,y) lying on the curve defined by the equation $g(x,y) = 0$ where $g(x,y) = y^3 - x^2$. Guesses do not suffice here!