Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Generic vector objects must be denoted by using arrows. Since the answer really consists of all the magic transformations, do not "box" your final results. Show me all the magic on the page neatly.

Silly 10 Point Bonus: Reveal how one can obtain the exact value of the definite integral

$$\iint_R e^{-(x^2+y^2)}\,dA$$

where R = { (x,y): $x \geq 0$ and $y \geq 0$ }, even though there is no elementary anti-derivative for the function

$$f(x) = e^{-x^2}.$$

Say where your work is, for it won't fit here! [You may gloss some of the technical details related to the matter of convergence.]

Of course, of you read with care Example 5 of Section 13.4 (Section 14.4 of the paperback version), you would find this a snooze. Clearly the integral is improper. By converting to polar coordinates, we have

$$\begin{aligned}\iint_R e^{-(x^2+y^2)}\,dA &= \int_0^{\pi/2}\int_0^{\infty} e^{-r^2}\,r\,dr\,d\theta \\ &= \lim_{b\to\infty}\int_0^{\pi/2}\int_0^{b} e^{-r^2}\,r\,dr\,d\theta \\ &= \lim_{b\to\infty}\int_0^{b} e^{-r^2}\,r\,dr\cdot\int_0^{\pi/2} 1\,d\theta \\ &= \lim_{b\to\infty}\left(\frac{\pi}{2}\right)\left(\frac{1}{2}-\frac{e^{-b^2}}{2}\right) \\ &= \frac{\pi}{4}.\end{aligned}$$

There are a few technical things regarding convergence that really need the careful use of a couple of inequalities here, but we'll let them slide. One of the neat consequences of this foolishness is being able to deduce that

$$\int_0^{\infty} e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}.$$

Happy Fubini to you.//o.o.