Name:

STUDENT #: EXAM #:

Read Me First: Use correct notation when presenting your computations. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Vector objects must be denoted by using arrows. Since the answer really consists of all the magic incantations and transformations, do not "box" your final results. Show me all the magic on the page neatly.

1. (15 pts.) Let $f(x,y) = 4y^2 + 3x^2$.
(a) Compute the gradient of f at (1,-1). (b) Then use it to compute $D_{\mathbf{u}}f(1,-1)$, where $\mathbf{u} = <\cos(4\pi/3), \sin(4\pi/3)>$ is the unit vector that forms an angle of $4\pi/3$ with respect to the positive x-axis. (c) Finally, obtain a unit vector $\mathbf{v}_0$ so that $D_{\mathbf{v}}f(1,-1)$ is a maximum as $\mathbf{v}$ is allowed to range over all unit vectors in the plane.

$\nabla f(x,y)$ =

$\nabla f(1,-1)$ =

$\mathbf{u}$ =

$D_{\mathbf{u}}f(1,-1)$ =

$\mathbf{v}_0$ =

2. (15 pts.) Evaluate the following line integral, where C is the path from (-1,-1) to (1,1) along the curve $x = y^3$. [Hint: The field is not conservative. The path is not closed. Parameterize C first. A picture might help -- or not.]

$$\int_C (2xy)\,dx + (x - y)\,dy =$$

3. (15 pts.) A particle, starting at (0,0) moves to the point (1,1) along the curve $y = x^5$ and then returns to (0,0) along the parabola $y = x^2$. Use Green's Theorem to compute the work done on the particle by the force field

$$\boldsymbol{F}(x,y) = \left\langle 8xy, 4x^2 + 10x \right\rangle.$$

4. (15 pts.) Find an arc-length parameterization for the curve

$$\boldsymbol{r}(t) = \left\langle \frac{1}{3}t^3, \frac{1}{2}t^2 \right\rangle, \quad t \geq 0,$$

that has the same orientation and has the point on the graph where $t = 0$ as the reference point. Rather than overloading the symbol **r**, write this new parameterization as **R**(s). If you move along the curve using the parameterization given by **R**, what's your speed?

5. (15 pts.) Evaluate the iterated integral

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \cos(x^2+y^2)\ dx\,dy$$

by first converting the integral to an equivalent integral in polar coordinates, and then evaluating the iterated integral you obtain. A picture of the region helps.

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \cos(x^2+y^2)\ dx\,dy \ =$$

6. (15 pts.) Write down but do not attempt to evaluate iterated triple integrals in (a) rectangular, (b) cylindrical, and (c) spherical coordinates that would be used to compute the volume of the sphere defined by $x^2 + y^2 + z^2 = (5)^2$. [For rectangular, there are many correct variants.]

(a) V =

(b) V =

(c) V =

7. (15 pts.) Determine the absolute maximum and minimum values of $f(x,y) = x^2 + 2y^2 - x$ and where they are obtained when (x,y) lies in the closed disk defined by $x^2 + y^2 \le 4$. Analyze f on the boundary by using Lagrange Multipliers, but don't forget the interior.

8. (15 pts.) (a) (12 pts.) Compute the unit vectors $\mathbf{T}(t)$ and $\mathbf{N}(t)$ and the curvature $\kappa(t)$ for the helix defined by

$$\mathbf{r}(t) = \langle \cos(t), t, \sin(t) \rangle.$$

(b) (3 pts.) Locate the center, $C(x_0,y_0,z_0)$, of the circle of curvature when $t = \pi/2$.

9. (15 pts.) Obtain a set of parametric equations for the line through the point (0, 0, 0) that is perpendicular to the plane that contains the three points (1,0,0), (0,2,0), and (0,0,3). Then locate the point in 3-space where the line intersects the plane.

10. (15 pts.) Locate and classify the critical points of the function $f(x,y) = x^3 - 3xy - y^3$. Use the second partials test in making your classification. (Fill in the table below after you locate all the critical points. It always helps to factor. How does a product equal zero??)

Crit.Pt.	f_{xx} @ cp	f_{yy} @ cp	f_{xy} @ cp	D @ cp	Conclusion

11. (10 pts.) Let $\boldsymbol{F}(x,y) = < 6x + y,\ 4y + x >$.

(a) Show that the field is conservative by producing a potential function $\phi(x,y)$ so that $\nabla\phi(x,y) = \mathbf{F}(x,y)$ for all (x,y) in the plane.

(b) Working within the influence of the force field from part (a), you move a particle along an ellipse C given by

$$\boldsymbol{r}(t) = < 3\cos(t)\,,\, 4\sin(t) >$$

from time $t_0 = \pi/2$ to time $t_1 = \pi$. How much work did you do??

12. (10 pts.)

Let $\boldsymbol{F}(x,y,z) = < x^2y\ ,\ 2y^3z\ ,\ 3z >$. Compute the divergence and the curl of the vector field $\mathbf{F}$.

(a) div $\boldsymbol{F}$ =

(b) curl $\boldsymbol{F}$ =

13. (10 pts.) (a) Obtain an equation for the plane that is tangent to the graph of the function $f(x,y) = x^2 + y^2$ at the point (1,2,5) in 3-space.

(b) Does the plane in part (a) that is tangent at (1,2,5) to the graph of $f(x,y) = x^2 + y^2$ intersect the yz-plane? If the answer is "yes", obtain a set of parametric equations in 3-space for the line of intersection.

14. (10 pts.) Evaluate the surface integral,

$$\iint_{\sigma} f(x,y,z)\ dS$$

where $f(x,y,z) = x - y - z$ and σ is the portion of the plane $x + y = 1$ in the first octant between $z = 0$ and $z = 1$.

$$\iint_{\sigma} f(x,y,z)\ dS =$$

15. (10 pts.) Let R be the triangular region enclosed by the lines $y = 0$, $y = x$, and $x + y = \pi/4$. Use a suitable transformation to compute the following integral:

$$\iint_R \frac{\sin(x-y)}{\cos(x+y)}\, dA =$$

Silly 10 Point Bonus: It turns out that routine computations reveal that

$$\int_0^\infty e^{-xy}\, dy = \frac{1}{x}$$

for $x > 0$ and

$$\int_0^\infty \sin(x) e^{-xy}\, dx = \frac{1}{1+y^2}$$

for $y > 0$. One interesting consequence is that, although there is no elementary antiderivative for the function $f(x) = \sin(x)/x$, one can completely evaluate the following improper integral:

$$\int_0^\infty \frac{\sin(x)}{x}\, dx.$$

Show how this is done.
Note: There is no problem with f at $x = 0$ since we "plug the hole" by setting $f(0) = 1$, the obvious limit value. Say where your work is for it won't fit here.