**Read Me First:** Show all essential work very neatly. Use correct notation when presenting your computations. Write using complete sentences. Remember this: "=" denotes "equals" , "$\Rightarrow$" denotes "implies" , and "$\Leftrightarrow$" denotes "is equivalent to". *Vector objects must be denoted by using arrows.* Since a correct answer really consists of all the magic transformations, do not "box" your final results. Show me all the magic on the page clearly.

1. (5 pts.) Obtain parametric equations for the line through the points (4,4,0) and (1,0,-1).

Using **v** = <-3,-4,-1> for our direction vector and the point (4,4,0) to build a set of parametric equations yields the following: $x = 4 - 3t$, $y = 4 - 4t$, and $z = -t$. There are, of course, infinitely many sets of equations that you can build.

2. (5 pts.) Obtain a vector equation for the line through (-1,-2, 3) and parallel to the line defined by $\langle x,y,z\rangle = \langle 2\pi,-\pi,3\pi\rangle + t\langle -6,0,-10\rangle$, where t is any real number.

$$\langle x,y,z\rangle = \langle -1,-2,3\rangle + t\langle -6,0,-10\rangle$$

is the most obvious solution.

3. (5 pts.) Obtain parametric equations for the line obtained when the two planes defined by $x + 3y + 2z = 5$ and $x + 2y + z = 10$ intersect.

Simply solving the system and using z as the free variable yields the following: $x = 20 + t$, $y = -5 - t$, and $z = t$. Another common answer is $x = 15 - t$, $y = t$, and $z = - 5 - t$. Do you see how these are related??

4. (5 pts.) Where does the line defined by the vector equation $\langle x,y,z\rangle = \langle 12,-12,6\rangle + t\cdot\langle 4,3,2\rangle$ intersect the yz-plane??

Recall that an equation for the yz-plane is $x = 0$. If $(x_0,y_0,z_0)$ is the point of intersection, then there is a number $t_0$ so that $12 + 4t_0 = x_0 = 0$ since $(x_0,y_0,z_0)$ lies on both the plane and the line. Solving this linear thing yields $t_0 = -3$. Consequently, $(x_0,y_0,z_0) = (0,-21,0)$.

5. (5 pts.) What point $(x_0,y_0)$ is seven-tenths of the way from P = (-2,-3) to Q = (48,7) ??

The vector **v** with initial point P and terminal point Q has coordinates <50,10> in standard position. Consequently, the terminal point of $\langle x_0,y_0\rangle = \langle -2,-3\rangle + (7/10)\langle 50,10\rangle = \langle 33,4\rangle$ in standard position provides us with the coordinates for $(x_0,y_0)$. Vector magic!!

6. (5 pts.) Suppose **v** = <-3,-2, 1> and **w** = <-1,2,4>. Then

$\mathbf{v}\bullet\mathbf{w} = (-3)(-1) + (-2)(2) + (1)(4) = 3 - 4 + 4 = 3$

7. (5 pts.) Suppose **v** = <-3,-2, 1> and **w** = <-2,2,1>. Then

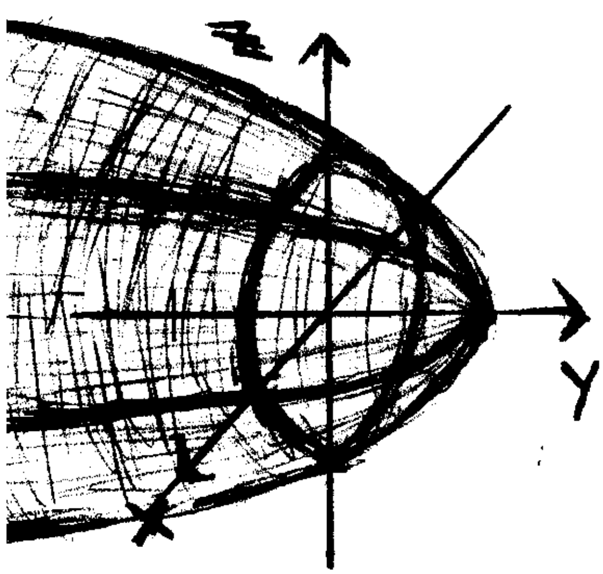
$\mathbf{v} \times \mathbf{w} = \langle -3,-2, 1\rangle \times \langle -2,2,1\rangle = \langle -4, -(-1), -10\rangle$

8. (5 pts.) Suppose **v** = <-3,-2, 1> and **w** = <-1,2,4>. Then

$\mathrm{proj}_{\mathbf{w}}(\mathbf{v}) = \langle -1/7,2/7,4/7\rangle$ , and the component of **v**

perpendicular to **w** is $\mathbf{w}_2 = \langle -20/7,-16/7,3/7\rangle$ .

Here is the back of Page 4 of 5:



9. (5 pts.) Suppose **v** = <-3,-2, 1> and **w** = <-2,2,1>.
If $\alpha$, $\beta$, and $\gamma$ are the direction angles of **w**, then

$\cos(\alpha) = -2/(9)^{1/2} = -2/3$ ,

$\cos(\beta) = 2/(9)^{1/2} = 2/3$ ,and

$\cos(\gamma) = 1/(9)^{1/2} = 1/3$ .

10. (5 pts.) Suppose **v** = <-3,-2, 1> and **w** = <-1,2,4>. What is the exact value of the angle $\theta$ between **v** and **w** ??

$\theta = \cos^{-1}((\mathbf{v}\bullet\mathbf{w})/(\|\mathbf{v}\|\cdot\|\mathbf{w}\|)) = \cos^{-1}(\ 3/(\ (14)^{1/2}\cdot(21)^{1/2}\ )$

$= \cos^{-1}(\ 3/(294)^{1/2}\ ) = \cos^{-1}(\ 3/[7(6)^{1/2}]\ )$

11. (5 pts.) Write a point-normal equation for the plane perpendicular to **v** = <-3,-2,1> and containing the point $(3\pi,2\pi,\pi)$.

$-3(x - 3\pi) - 2(y - 2\pi) + (z - \pi) = 0$

12. (5 pts.) Write an equation for the plane passing through the point (4,-5, 6) and perpendicular to the line defined by the vector equation $<x,y,z> = <4e,25,2\pi> + t<2,-3,-3>$.

The direction vector in the equation for the line provides us with a normal vector for the plane because the line is to be perpendicular to the plane. Consequently, a simple point-normal equation for the plane is given by

$2(x-4) - 3(y+5) - 3(z-6) = 0.$

13. (5 pts.) Find the exact value of the acute angle θ of intersection of the two planes defined by the two equations

$$x - 2y = -55 \quad \text{and} \quad 3y - 4z = 75.$$

$$\theta = \cos^{-1}((|\mathbf{v}\bullet\mathbf{w}|)/(\|\mathbf{v}\|\cdot\|\mathbf{w}\|)) = \cos^{-1}(6/(125)^{1/2})$$

where **v** = <1,-2,0> and **w** = <0,3,-4>. Observe acutely the funny absolute value thingies. Why do we need them generally to get acute angles??

14. (5 pts.) Write an equation for the plane which contains the line defined by <x,y,z> = <1,2,3> + t<3, -2, 1> and is perpendicular to the plane defined by x - 2y + z = 0.

Since a normal vector **v** for the plane must be perpendicular to both the direction of the line, due to the line lying in the desired plane, and any non-zero normal vector for the given plane defined by the equation x - 2y + z = 0, we may build **v** by means of a cross product. Set **v** = <3,-2,1> × <1,-2,1> = <0,-2,-4>. A point-normal equation for the plane we want is now cheap thrills: -2(y - 2) - 4(z - 3) = 0 ... ugh.

15. (5 pts.) What is the radius of the sphere centered at (1, 0, 0) and tangent to the plane defined by x + 2y + z = 10?

The radius,r , of the sphere will simply be the distance from the point (1,0,0) to the plane defined by the equation x - 2y + z - 10 = 0. Using the appropriate formula, we have

$$r = |(1)(1)+(2)(0)+(1)(0)-10|/(1 + 4 + 1)^{1/2} = 9/(6)^{1/2}.$$

16. (5 pts.) The equation

$$z = 3r^2\cdot\cos^2(\theta)$$

is in cylindrical coordinates. Obtain an equivalent equation in terms of rectangular coordinates (x,y,z).

Wake me up ... $z = 3x^2$.

17. (5 pts.) The point $(5,-5\sqrt{3},10)$ is in rectangular coordinates. Convert this to spherical coordinates $(\rho,\theta,\phi)$.

$(\rho,\theta,\phi) = ( 10\cdot(2)^{1/2} , 5\pi/3, \pi/4)$ .

$\rho^2 = 5^2 +(-5\sqrt{3})^2 + 10^2 = 200$

$\phi = \cos^{-1}(1/(2)^{1/2}) = \pi/4$

$\theta$ has its terminal side in the 4th quadrant, and its reference angle is $\theta_r = \tan^{-1}(\sqrt{3}) = \pi/3$. Consequently $\theta = 2\pi - \theta_r = 5\pi/3$.

18. (5 pts.) Do the lines defined by the equations

$$\langle x,y,z\rangle = \langle 3,1,2\rangle + t\langle 2,-1,-2\rangle$$

and

$$\langle x,y,z\rangle = \langle 10,8,-5\rangle + t\langle 1,3,-1\rangle$$

intersect? If they do intersect, what is the point of intersection??

If the lines are to intersect, there must be numbers $t_1$ and $t_2$ so $3 + 2t_1 = 10 + t_2$, $1 - t_1 = 8 + 3t_2$, and $2 - 2t_1 = -5 - t_2$. [Here $t_1$ is the parameter for the putative point in terms of the first equation and $t_2$ is the parameter value for the second equation.] Solving this system yields $t_1 = 2$ and $t_2 = -3$. Using either $t_1$ or $t_2$ in the appropriate vector equation yields the point of intersection, $(7,-1,-2)$.

19. (5 pts.) What is the area of the triangle in three space with vertices at $P = (1, 0, 0)$, $Q = (0, 2, 0)$, and $R = (0 , 0, 3)$.

Let **v** be the vector with initial point P and terminal point Q, and let **w** be the vector with initial point P and terminal point R. Then the area A of the triangle is given by
$A = \|\mathbf{v} \times \mathbf{w}\|/2 = \|\langle -1,2,0\rangle \times \langle -1,0,3\rangle\|/2 = \|\langle 6,3,2\rangle\|/2 = 7/2$.

20. (5 pts.) Do the three 2-space sketches of the traces in each of the coordinate planes of the surface defined by $y = 1 - x^2 - z^2$. Work below and label carefully. Then attempt to do a 3 - space sketch in the plane of the surface on the back of page 4.