Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Vector objects must be denoted by using arrows. Do not "box" your final results. Show me all the magic on the page.

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1. (10 pts.) Suppose that an object is moving in a fixed plane with its acceleration given by **a**(t) = <0,-32>. Suppose that the initial position of the object is **r**(0) = <1,1> and the initial velocity of the object is **v**(0) = < -2, 2>.

(a) (6 pts.) Find the velocity of the object, **v**(t), as a function of time.

(b) (2 pts.) Find the position of the object, **r**(t), as a function of time.

(c) (2 pts.) Obtain an equation for the parabola that is the path of the object.

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2. (10 pts.) Let **r**(t) = < 4cos(t) , 4sin(t), 3t > for $t \in \mathbb{R}$. Then

(a) **r**′(t) = ,

(b) **r**″(t) = ,

(c) **T**(t) = ,

(d) **N**(t) = , and

(e) $\kappa(t)$ = .

3. (10 pts.) Obtain an arc-length parameterization for the curve **r**(t) = < t, 3cos(t), 3sin(t) > in terms of the initial point (0, 3, 0) which is the terminal point of **r**(0) in standard position. Rather than overloading the symbol **r**, write this new parameterization as **R**(s). How are **R** and **r** related?

4. (10 pts.) A particle moves smoothly in such a way that at a particular time t = 0, we have **v**(0) = < 0 ,-4> and **a**(0) = < 2 , 3>. If we write **a**(0) in terms of **T**(0) and **N**(0), then

$$\mathbf{a}(0) = a_T(0)\mathbf{T}(0) + a_N(0)\mathbf{N}(0),$$

where

(a) **T**(0) = ,

(b) $a_T(0)$ = ,

(c) $a_N(0)$ = , and

(d) **N**(0) = .

(e) Also, $\kappa(0)$ =

5. (10 pts.) (a) Find the limit.

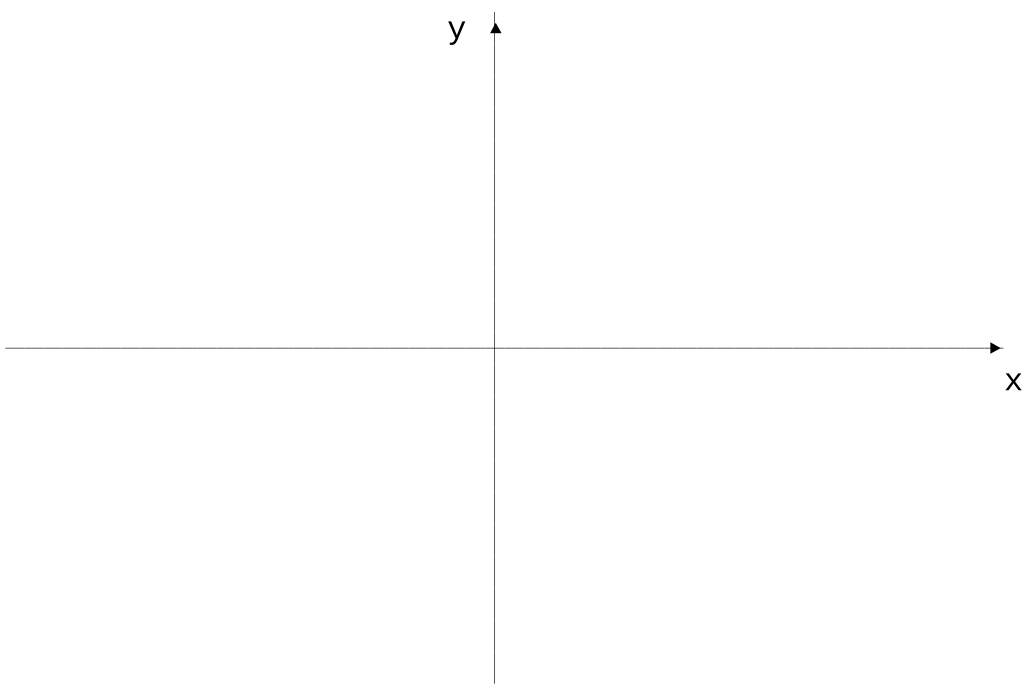
$$\lim_{t \to \infty} \left\langle \frac{t^2+1}{3t^2+2}, \frac{1}{t} \right\rangle =$$

(b) Find parametric equations for the line tangent to the graph of $\mathbf{r}(t) = t^2\mathbf{i} + (2 - \ln(t))\mathbf{j}$ at the point where $t_0 = 1$.

6. (10 pts.) Let

$$\boldsymbol{r}(t) = < 2 + 2\cos(t), -2\sin(t) >$$

for $\pi/2 \le t \le 3\pi/2$. (a) Sketch the curve defined by this vector-valued function by eliminating the parameter t from the components x and y. Label very carefully. (b) Indicate the direction of increasing t. (c) Give the curvature $\kappa(t_0)$ of the curve at $\mathbf{r}(t_0)$ for each t_0. [Warning: Pythagoras is perched on your shoulder, but be careful of applying too much power.]



7. (10 pts.) Let $f(x,y) = xy + 3$.

(a) Find $f(x + y\ ,\ x - y)$.

$f(x + y\ ,\ x - y) =$

(b) Find an equation of the level curve that passes through the point (-1,2).

8. (10 pts.). (a) Use limit laws and continuity properties to evaluate the following limit.

$$\lim_{(x,y)\to(1/2,\pi)} (xy^2\sin(xy)) =$$

(b) Evaluate the limit, if it exists, by converting to polar coordinates.

$$\lim_{(x,y)\to(0,0)} \frac{\tan(\pi(x^2+y^2))}{x^2+y^2} =$$

9. (10 pts.) (a) Calculate $\partial z/\partial y$ using implicit differentiation when $(x^2 + y^2 + z^2)^{3/2} = 1$. Leave your answer in terms of x, y, and z.

(b) Find all second-order partial derivatives for the function $f(x,y) = x^2y^3$. Label correctly.

10. (10 pts.) (a) Compute the differential dz when

$$z = \tan^{-1}(xy).$$

$dz =$

(b) Let $f(x,y) = \ln(xy)$. Find the local linear approximation L to f at the point $P(1,2)$.

$L(x,y) =$

Silly 10 Point Bonus: Let g be defined by

$$g(x,y) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0). \end{cases}$$

Explicitly compute $g_{xy}(0,0)$ and $g_{yx}(0,0)$. [Say where your work is, for it won't fit here.]