**Read Me First:** Show all essential work very neatly. Use correct notation when presenting your computations. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Generic vector objects must be denoted by using arrows. Since the answer really consists of all the magic transformations, do not "box" your final results. Show me all the magic on the page neatly.

1. (10 pts.) Obtain an equation for the plane tangent to the graph of $f(x,y) = \sec(xy)$ when $(x_0,y_0) = (\pi,1/4)$. Then obtain a vector equation for the normal line to the surface at the same point. [ **Reminder**: $z_0 = f(x_0,y_0)$ ]

2. (10 pts.) Let

$$f(x,y) = (y^2 - x^2)^{1/3}.$$

Compute the gradient of $f$ at $(-1,3)$. Then use it to compute the directional derivative $D_{\mathbf{u}}f(-1,3)$, where $\mathbf{u}$ is the unit vector forming an angle of $\theta = 5\pi/6$ with respect to the positive x-axis.

$\nabla f(x,y) =$

$\nabla f(-1,3) =$

$\boldsymbol{u} =$

$D_{\mathbf{u}}f(-1,3) =$

3. (10 pts.) Suppose $f(x,y) = x^2y$. Find all unit vectors **u** so that

$$(*) \quad D_{\mathbf{u}}f(-1,2) = 0$$

is true for **u**.

4. (10 pts.) Assume $f(1,-2) = 4$ and $f(x,y)$ is differentiable at $(1,-2)$ with $f_x(1,-2) = 2$ and $f_y(1,-2) = -3$. Using an appropriate local linear approximation, estimate the value of $f(0.9,-1.950)$.

$f(0.9,-1.950) \approx$

5. (10 pts.) (a) Use an appropriate form of chain rule to find $\partial z/\partial v$ when $z = \cos(x)\sin(y)$ when $x = u - v$ and $y = u^2 + v^2$.

$$\frac{\partial z}{\partial v} =$$

(b) Assume that $F(x,y,z) = 0$ defines $z$ implicitly as a function of $x$ and $y$. Show that if $\partial F/\partial z \neq 0$, then

$$\frac{\partial z}{\partial y} = -\frac{\partial F/\partial y}{\partial F/\partial z}.$$

6. (10 pts.) Locate all relative extrema and saddle points of the following function.

$$f(x,y) = x^4 + y^4 - 4xy$$

Use the second partials test in making your classification. (Fill in the table below after you locate all the critical points.)

| Crit.Pt. | $f_{xx}$ @ c.p. | $f_{yy}$ @ c.p. | $f_{xy}$ @ c.p. | D @ c.p. | Conclusion |
|---|---|---|---|---|---|
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**7. (10 pts.)** (a) Evaluate the following iterated integral.

$$\int_0^{\ln(3)}\int_0^{\ln(2)} e^{x+y}\,dy\,dx =$$

(b) Very carefully write down, *but do not evaluate*, an iterated integral whose numerical value is the volume under the plane $z = 2x + y$ and over the rectangle $R = \{(x,y) : 3 \le x \le 5,\ 1 \le y \le 2\}$.

$$V =$$

**8. (10 pts.)** Evaluate the integral below by first reversing the order of integration. Sketching the region is a key piece of the puzzle.

$$\int_0^{4}\int_{\sqrt{y}}^{2} e^{x^3}\,dx\,dy =$$

9. (15 pts.) Let $f(x,y) = x^2 - y^2$ on the closed unit disk defined by $x^2 + y^2 \leq 1$. Find the absolute extrema and where they occur. Use Lagrange multipliers to analyze the function on the boundary.

10. (5 pts.) Find a unit vector in the direction in which $f(x,y)$ increases most rapidly at $P_0 = (0,\pi/6)$ when

$$f(x,y) = \cos(3x - y).$$

**Silly 10 Point Bonus:** (a) State the definition of differentiability for a function of two variables. [You may either state the definition found in the text or the one given by the instructor in class.] (b) Then using only the definition you stated, show the function

$$f(x,y) = xy$$

is differentiable at any point (x,y) in the plane. Say where your work is, for it won't fit here.