Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Generic vector objects must be denoted by using arrows. Since the answer really consists of all the magic transformations, do not "box" your final results. Show me all the magic on the page neatly.

1. (10 pts.)
Let **F**(x,y,z) = <x^2, -2, yz>. Compute the divergence and the curl of the vector field **F**.

(a) div **F** =

(b) curl **F** =

2. (10 pts.) Convert the given iterated integral into an iterated integral in polar coordinates that has the same numerical value and is easier to evaluate, perhaps. Do not attempt to evaluate the iterated integrals. A picture might help.

$$\int_{-3}^{0}\int_{0}^{(9-x^2)^{1/2}} 2x\,(x^2+y^2)\;dy\,dx \;=$$

3. (10 pts.) Write down the triple iterated integral in cylindrical coordinates that would be used to compute the volume of the solid G whose top and bottom are on the sphere defined by the equation $r^2 + z^2 = 100$ and whose lateral boundary is given by the cylinder defined by the equation $r = 5\cdot\sin(\theta)$. Do not attempt to evaluate the integral. (**WARNING:** $r = 5\cdot\sin(\theta)$ has teeth. If you don't pay attention to the details, it bites.)

$$\iiint_G 1\, dV =$$

4. (10 pts.) Write down a triple iterated integral in cartesian coordinates that would be used to find the volume of the solid G bounded by the surface $y = x^2$ and the planes $y + z = 4$ and $z = 0$, but do not attempt to evaluate the triple iterated integral you have obtained. [Sketching the traces in the coordinate planes will help.]

$$\iiint_G 1\, dV =$$

5. (10 pts.) Write down the triple iterated integral in spherical coordinates that would be used to compute the volume of the solid *G* within the cone defined by $\phi = \pi/4$ and between the spheres defined by $\rho = 4$ and $\rho = 9$ *in the first octant*. Do not attempt to evaluate the interated integrals.

$$\iiint_G 1 \, dV =$$

6. (10 pts.) Compute the surface area of the portion of the hemisphere defined by $z = (16 - x^2 - y^2)^{1/2}$ that lies between the planes defined by $z = 1$ and $z = 2$.

7. (10 pts.) Compute the work in moving a particle along of the path in the xy-plane that goes from the point (-1,-1) to the point (1,1) along the curve $y = x^5$ against the force field defined by

$$\mathbf{F}(x,y) = \langle\, y-x \;,\; yx^3 \,\rangle.$$

$W =$

8. (10 pts.) Starting at the point (0,0), a particle goes along the y-axis until it reaches the point (0,4). It then goes from (0,4) to (-4,0) along the circle with equation $x^2 + y^2 = 16$. Finally the particle returns to the origin by travelling along the x-axis. Use Green's Theorem to compute the work done on the particle by the force field defined by $\mathbf{F}(x,y) = < -5y^3, 5x^3 >$ for $(x,y)\ \varepsilon\ \mathbb{R}^2$. [Draw a picture. This is easy??]

9. (10 pts.) (a) Show that the vector field

$$\mathbf{F}(x,y) = < \cos(x)e^{y} , \sin(x)e^{y} + 10y >$$

is actually a gradient field by producing a function $\phi(x,y)$ such that $\nabla\phi(x,y) = \mathbf{F}(x,y)$ for all (x,y) in the plane.

(b) Using the Fundamental Theorem of Line Integrals, evaluate the path integral below, where C is any smooth path from the origin to the point $(\pi/2,\ln(2))$. [**WARNING:** You must use the theorem to get any credit here.]

$$\int_{C} (\cos(x)e^{y})\,dx + (\sin(x)e^{y} + 10y)\,dy =$$

10. (10 pts.) Use the substitution $u = (1/2)(x + y)$ and $v = (1/2)(x - y)$ to evaluate the integral below, where R is the bound region enclosed by the triangular region with vertices at (0,0), (2,0), and (1,1).

$$\iint_R \cos\left(\frac{1}{2}(x+y)\right)\, dA = \iint_R \cos\left(\frac{1}{2}(x+y)\right)\, dA_{x,y}$$

$$=$$

Silly 10 Point Bonus: Become a polar explorer. Reveal the details of how one can obtain the exact value of the definite integral

$$\int_0^\infty e^{-x^2}\, dx$$

even though there is no elementary anti-derivative for the function

$$f(x) = e^{-x^2}.$$

Say where your work is, for it won't fit here! [You may gloss some of the technical details related to the matter of convergence.]