STUDENT #: EXAM #:

Read Me First: *Read each problem carefully and do exactly what is requested. Full credit will be awarded only if you show all your work neatly, and it is correct. Use complete sentences and use notation correctly. Remember that what is illegible or incomprehensible is worthless. Communicate. Eschew obfuscation. Good Luck!*

1. (12 pts.) Evaluate the iterated integral by reversing the order of integration.

$$\int_0^4 \int_{\sqrt{y}}^2 e^{x^3}\, dx\, dy =$$

2. (18 pts.) (a) Obtain an equation for the plane that is tangent to the graph of the function

$$f(x,y) = x^2 + y^2$$

at the point (-1,-2,5) in 3-space.

(b) Consider the function f of three variables defined by

$$f(x,y,z) = 5x^2 + 6y^2 - 4z^2.$$

Obtain an equation for the level surface for this function that passes through the point (1,-1, -1) in 3-space.

(c) Obtain an equation for the plane that is tangent to the level surface given in part (b) with the point of tangency being (1,-1, -1).

3. (12 pts.) Obtain an arc-length parameterization for the curve

$$\boldsymbol{r}(t) = \langle \cos(e^t), e^t, \sin(e^t) \rangle \quad for\ t \geq 0.$$

in terms of the initial point (cos(1), 1, sin(1)) which is the terminal point of $\boldsymbol{r}(0)$ in standard position. Rather than overloading the symbol $\boldsymbol{r}$, write this new parameterization as $\boldsymbol{R}(s)$. How are $\boldsymbol{R}$ and $\boldsymbol{r}$ related?

4. (16 pts.) Locate and classify the critical points of the function

$$f(x,y) = 2x^3 - 6xy + y^2$$

Use the second partials test in making your classification.

Crit.Pt.	f_{xx} @ c.p.	f_{yy} @ c.p.	f_{xy} @ c.p.	D @ c.p.	Conclusion

5. (16 pts.) Find the absolute extrema of $f(x,y) = x^2 + 2y^2 - x$ in the region

$D = \{(x,y) : x^2 + y^2 \le 4\}$.

6. (12 pts.) Evaluate the following line integral, where C is the path from (1,-1) to (1,1) along the curve $x = y^4$.

$$\int_C (y - x)\,dx + (2xy)\,dy =$$

7. (10 pts.) (a) Show that the vector field

$$\mathbf{F}(x,y) = < \cos(x)e^{y} + 2x \ , \ \sin(x)e^{y} >$$

is actually a gradient field by producing a function $\phi(x,y)$ such that $\nabla\phi(x,y) = \mathbf{F}(x,y)$ for all (x,y) in the plane.

(b) Using the Fundamental Theorem of Line Integrals, evaluate the path integral below, where C is any smooth path from the origin to the point $(\pi/2, \ln(2))$. [**WARNING:** You must use the theorem to get credit here.]

$$\int_{C} (\cos(x)e^{y} + 2x)\,dx + (\sin(x)e^{y})\,dy \ =$$

8. (18 pts.) Write down but do not attempt to evaluate the iterated triple integrals in (a) rectangular, (b) cylindrical, and (c) spherical coordinates that would be used to compute the volume of the sphere with a radius of 1 centered at the origin. [For rectangular, there are many correct variants.]

(a) V =

(b) V =

(c) V =

9. (12 pts.) Let

$$\boldsymbol{F}(x,y,z) = \langle yz, xy^2, yz^2 \rangle \ .$$

Compute the divergence and the curl of the vector field **F**.

(a) div ***F*** =

(b) curl ***F*** =

10. (16 pts.) Use the substitution $u = x + y$, $v = x - y$ to evaluate the integral

$$\iint_R (x-y)\, e^{x^2-y^2}\, dA$$

where R is the region enclosed by the lines $x + y = 0$, $x + y = 1$, $x - y = 1$, and $x - y = 4$

11. (12 pts.) A particle, starting at (0,0) moves to the point (1,1) along the parabola $y = x^2$ and then returns to (0,0) along the parabola $x = y^2$. Use Green's Theorem to compute the work done on the particle by the force field

$$\boldsymbol{F}(x,y) = \left\langle 8xy, 4x^2 + 10x \right\rangle.$$

12. (16 pts.) Evaluate the surface integral,

$$\iint_{\sigma} f(x,y,z)\ dS$$

where $f(x,y,z) = x^2z$ and σ is the portion of the cone $z = (x^2 + y^2)^{1/2}$ between between the planes $z = 1$ and $z = 2$.

$$\iint_{\sigma} f(x,y,z)\ dS \ =$$

13. (12 pts.) (a) If f is differentiable at (x_0,y_0), then the local linear approximation to f at (x_0,y_0) is

$L(x,y)$ =

(b) Assume $f(1,-2) = 4$ and $f(x,y)$ is differentiable at (1,-2) with $f_x(1,-2) = 2$ and $f_y(1,-2) = -3$. Using an appropriate local linear approximation, estimate the value of $f(0.9,-1.950)$.

$f(0.9,-1.950) \approx$

14. (18 pts.) Compute the unit vectors **T**(t) and **N**(t) and the curvature $\kappa(t)$ for the "hot-rod" helix defined by

$$\boldsymbol{r}(t) = \langle \cos(e^t), e^t, \sin(e^t) \rangle$$

Silly 10 Point Bonus: Show how to use the Fundamental Theorem of Calculus to prove the Fundamental Theorem of Line Integrals. Say where your work is, for it won't fit here. [If you intend to seriously attempt this, you might begin by actually stating the Fundamental Theorem of Line Integrals.]