**Read Me First:** Show all essential work very neatly. Use correct notation when presenting your computations. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". *Vector objects must be denoted by using arrows.* Since a correct answer really consists of all the magic transformations, do not "box" your final results. Show me all the magic on the page clearly. Eschew Obfuscation.

1. (5 pts.) Obtain parametric equations for the line through the points (-4,-4,1) and (2,0,2).

Using **v** = <6,4,1> for our direction vector and the point (-4,-4,1) to build a set of parametric equations yields the following: x = -4 + 6t, y = -4 + 4t, and z = 1 + t. There are, of course, infinitely many sets of equations that you can build that yield this line.

2. (5 pts.) Obtain a vector equation for the line through (-1,-2, 3) and parallel to the line defined by <x,y,z> = <-6,0,-10> + t<2,-π,3>, where t is any real number.

$$\langle x,y,z\rangle = \langle -1,-2,3\rangle + t\langle 2,-\pi,3\rangle$$

is the most obvious solution.

3. (5 pts.) Obtain parametric equations for the line obtained when the two planes defined by x + y + 2z = 5 and x - 2y + z = 10 intersect.

Simply solving the system and using z as the free variable yields the following:

$$\begin{cases} x = \frac{20}{3} - \frac{5}{3}t \\ y = -\frac{5}{3} - \frac{1}{3}t \\ z = t \qquad , t \in \mathbb{R} \end{cases}$$

Other answers may be checked by ensuring direction vectors are parallel and that the "chosen point" lives here as well!!

4. (5 pts.) What is the radius of the sphere with center (1,1,1) that is tangent to the plane defined by x + 2y + 2z = 11 and what is the point of tangency?

The radius is simply the distance from the point (1,1,1) to the plane, namely

$$r = \frac{|(1)+2(1)+2(1)-11|}{\sqrt{(1)^2+(2)^2+(2)^2}} = \ldots = 2.$$

The tangent point is the point of intersection of the plane and the line through the point (1,1,1) and perpendicular to the plane with vector equation

$$\langle x,y,z\rangle = \langle 1,1,1\rangle + t\langle 1,2,2\rangle \text{ , } for\ t \in \mathbb{R}.$$

The parameter value t for the point of intersection satisfies the equation

$$(1+t)+2(1+2t)+2(1+2t) = 11 \quad or \quad t = \frac{2}{3}.$$

Substituting this into the equation for the line reveals that the point has coordinates (*x*,*y*,*z*) = (5/3,7/3,7/3)

5. (5 pts.) What point $(x_0,y_0)$ is four-fifths of the way from P = (-2,-3) to Q = (48,7) ??

The vector **v** with initial point P and terminal point Q has coordinates <50,10> in standard position. Consequently, the terminal point of $\langle x_0,y_0\rangle = \langle -2,-3\rangle + (4/5)\langle 50,10\rangle = \langle 38,8\rangle$ in standard position provides us with the coordinates for $(x_0,y_0)$. Vector magic!!

6. (5 pts.) Suppose **v** = <-3,-2, 1> and **w** = <-1,1,1>. Then

**v•w** = (-3)(-1) + (-2)(1) + (1)(1) = 3 - 2 + 1 = 2

7. (5 pts.) Suppose **v** = <-3,-2, 1> and **w** = <-1,1,1>. Then

**v** × **w** = < -3, -(2), -5 > = < -3, 2, -5 >

8. (5 pts.) Suppose **v** = <-3,-2, 1> and **w** = <-1,1,1>. Then

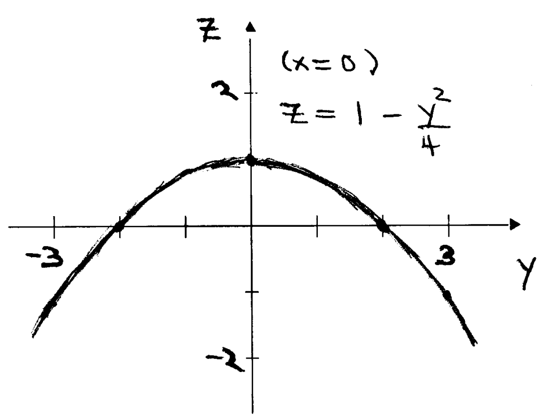
$\text{proj}_{\mathbf{w}}(\mathbf{v})$ = < -2/3, 2/3, 2/3 > ,

and the component of **v** perpendicular to **w** is

$\mathbf{w}_2$ = < -7/3, -8/3, 1/3 > .

The yz-plane part of Problem 20. (5 pts.) Do the three 2-space sketches of the traces in each of the coordinate planes of the surface defined by

$$z = 1 - \frac{x^2}{9} - \frac{y^2}{4} .$$



9. (5 pts.) Suppose **v** = <-3,-4, 5> and **w** = <-1,1,1>.
If $\alpha$, $\beta$, and $\gamma$ are the direction angles of **v**, then

$\cos(\alpha) = -\frac{3}{\sqrt{50}}$ ,

$\cos(\beta) = -\frac{4}{\sqrt{50}}$ ,and

$\cos(\gamma) = \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}}$ .

10. (5 pts.) Suppose **v** = <-3,-2, 1> and **w** = <-1,1,1>. What is the exact value of the angle $\theta$ between **v** and **w** ??

$$\theta = \cos^{-1}\left(\frac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\cdot\|\vec{w}\|}\right) = \cos^{-1}\left(\frac{2}{\sqrt{42}}\right)$$

11. (5 pts.) Write a point-normal equation for the plane perpendicular to **v** = <-3,-2,1> and containing the point (-1,2,-3).

$$-3(x - (-1)) - 2(y - 2) + (z - (-3)) = 0$$

12. (5 pts.) Which point on the line defined the vector equation <x,y,z> = <1,1,1> + t<2,-1,-1> is nearest the point, (0,-1,0)?

Build the vector with initial point (0,-1,0) to an arbitrary point on the line with parameter *t*,

$$\vec{v}(t) = \langle 1+2t, 2-t, 1-t\rangle.$$

The norm of this vector is the distance from the point (0,-1,0) to the point on the line with parameter value *t*. The norm will be smallest when the vector is perpendicular to the vector <2,-1,-1>. Consequently,

$$0 = \vec{v}(t)\cdot\langle 2,-1,-1\rangle = \langle 1+2t, 2-t, 1-t\rangle\cdot\langle 2,-1,-1\rangle = 6t - 1$$

provides us with the value of *t* needed for the closest point. The point in question may now be obtained using the vector equation for the line. It is (8/6,5/6,5/6).

13. (5 pts.) Find the exact value of the acute angle θ of intersection of the two planes defined by the two equations

$$x - 3y = -5 \quad \text{and} \quad 2y - 4z = 7.$$

$\theta = \cos^{-1}((|\mathbf{v}\bullet\mathbf{w}|)/(\|\mathbf{v}\|\cdot\|\mathbf{w}\|)) = \cos^{-1}(6/(200)^{1/2}) = \cos^{-1}(3/(5(2)^{1/2}))$

where **v** = <1,-3,0> and **w** = <0,2,-4>. Observe acutely the funny absolute value thingies.

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14. (5 pts.) Write an equation for the plane which contains the line defined by <x,y,z> = <1,2,3> + t<3, -2, 1> and is perpendicular to the plane defined by x - 2y + z = 0.

A normal vector ***n*** for the plane sought is

$$\vec{n} = \langle 3,-2,1\rangle \times \langle 1,-2,1\rangle = \langle 0,-2,-4\rangle$$

since it must be perpendicular to a direction vector for the line and a normal vector of the given plane. [One may also obtain a suitable vector by solving an appropriate system of equations, a little two by three linear homogeneous thingy.] An equation for the plane is now cheap thrills: $-2(y - 2) - 4(z - 3) = 0$, or equivalently, $y + 2z - 8 = 0$.

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15. (5 pts.) Obtain an equation for the plane tangent to the sphere defined by

$$(x-1)^2 + (y+2)^2 + (z-3)^2 = 9$$

at the point (2, -4, 5), which is actually on the sphere.

Here all we need is a normal vector for the tangent plane. This can be obtained easily using the point of tangency and the center of the sphere at hand.

$$\vec{n} = \langle 1-2,-2-(-4),3-5\rangle = \langle -1,2,-2.\rangle$$

An equation of the tangent plane:

$$-(x - 2) + 2(y - (-4)) - 2(z - 5) = 0.$$

You may obtain an equivalent standard form varmint if you wish.

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16. (5 pts.) The equation

$$r = 4\cos(\theta) - 10\sin(\theta)$$

is that of a cylinder in cylindrical coordinates. Obtain an equivalent equation in terms of rectangular coordinates (x,y,z). Provide a vector equation for the straight line that is the axis of symmetry.

Multiply the given equation by $r$. Then doing the usual conversion and completing the square a couple of times yields

$$(x-2)^2 + (y+5)^2 = 29.$$

A vector equation for the line of symmetry in 3-space is

$$\langle x,y,z\rangle = \langle 2,-5,t\rangle,\ \textit{for}\ t \in \mathbb{R}.$$

17. (5 pts.) The point (-3,-4,-5) is in rectangular coordinates. Convert this to spherical coordinates $(\rho,\theta,\phi)$. [Inverse trig fun?]

$$(\rho,\theta,\phi) = (\sqrt{50}, \pi + \tan^{-1}(\tfrac{4}{3}), \cos^{-1}(-\tfrac{5}{\sqrt{50}})) = (5\sqrt{2}, \pi + \tan^{-1}(\tfrac{4}{3}), \tfrac{3\pi}{4})$$

18. (5 pts.) Do the lines defined by the equations

$\langle x,y,z\rangle = \langle 0,1,2\rangle + t\langle 4,-2,2\rangle$ and $\langle x,y,z\rangle = \langle 1,1,-1\rangle + t\langle 1,-1,4\rangle$

intersect? Justify your answer, for *yes* or *no* does not suffice.

If the lines are to intersect, there must be numbers $t_1$ and $t_2$ so $4t_1 = 1 + t_2$, $1 - 2t_1 = 1 - t_2$, and $2 + 2t_1 = -1 + 4t_2$. [Here $t_1$ is the parameter for the putative point in terms of the first equation and $t_2$ is the parameter value for the second equation.] Solving this system yields $t_1 = 1/2$ and $t_2 = 1$. Using either $t_1$ or $t_2$ in the appropriate vector equation yields the point of intersection, (2, 0, 3). Note, however, you really do not have to obtain the point itself to answer this question!!

19. (5 pts.) What is the area of the triangle in three space with vertices at P = (-3, 0, 0), Q = (0, 4, 0), and R = (0 , 0, 4).

Let **v** be the vector with initial point P and terminal point Q, and let **w** be the vector with initial point P and terminal point R. Then the area A of the triangle is given by

$$A = \|\mathbf{v} \times \mathbf{w}\|/2 = \|\langle 3,4,0\rangle \times \langle 3,0,4\rangle\|/2$$

$$= \|\langle 16,-12,-12\rangle\|/2 = 4\|\langle 4,-3,-3\rangle\|/2 = 2\|\langle 4,-3,-3\rangle\|$$

$$= 2(34)^{1/2} = (544)^{1/2}/2$$ ???? regressing...???

20. (5 pts.) Do the three 2-space sketches of the traces in each of the coordinate planes of the surface defined by

$$z = 1 - \frac{x^2}{9} - \frac{y^2}{4}.$$

Do not attempt to do a 3 - space sketch. If you don't have enough space below, say where any additional work is.

The xy-plane and the xz-plane appear below. The yz-plane appears on Page 2 of 5.