Name:

Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". **Vector objects must be denoted by using arrows.** Do not "box" your final results. Show me all the magic on the page.

1. (10 pts.) Solve the following vector initial-value problem for $\mathbf{y}(t)$:

$$\mathbf{y}''(t) = \mathbf{i} + e^t\mathbf{j}\,,\ \mathbf{y}(0) = 2\mathbf{i}\,,\ \textit{and}\ \mathbf{y}'(0) = \mathbf{j}.$$

2. (10 pts.) Let $\mathbf{r}(t) = \langle 4\cos(t), 4\sin(t), t \rangle$.

Find $\mathbf{T}(\pi/2), \mathbf{N}(\pi/2),$ *and* $\kappa(\pi/2)$.

3. (10 pts.) Obtain an arc-length parameterization for the curve **r**(t) = < t, 3cos(2t), 3sin(2t) > in terms of the initial point (0, 3, 0) which is the terminal point of **r**(0) in standard position. Rather than overloading the symbol **r**, write this new parameterization as **R**(s). How are **R** and **r** related?

4. (10 pts.) A particle moves smoothly in such a way that at a particular time t = 0, we have **v**(0) = < 1 , 2 > and **a**(0) = < 3 , 0 >. If we write **a**(0) in terms of **T**(0) and **N**(0), then

$$\mathbf{a}(0) = a_T(0)\mathbf{T}(0) + a_N(0)\mathbf{N}(0),$$

where

(a) **T**(0) = ,

(b) $a_T(0)$ = ,

(c) $a_N(0)$ = , and

(d) **N**(0) = .

(e) Also, $\kappa(0)$ =

5. (10 pts.) (a) Find the limit.

$$\lim_{t \to 1} \left\langle \frac{3}{t^2}, \frac{\ln(t)}{t^2-1}, \sin(\frac{\pi}{2}t) \right\rangle =$$

(b) Find parametric equations for the line tangent to the graph of $\mathbf{r}(t) = (2 - \ln(t))\mathbf{i} + t^2\mathbf{j}$ at the point where $t_0 = 1$.

6. (10 pts.) Let

$$f(x,y) = y - x^2 .$$

(a) Obtain an equation for the level curve for f that passes through the point (-1,2).

(b) Compute the value of the directional derivative

$$D_{\mathbf{u}}f(-1,2)$$

when **u** is the unit vector in the plane that is in the direction of the gradient of f at (-1,2).

7. (10 pts.) (a) Use an appropriate form of chain rule to find $\partial z/\partial v$ when $z = \sin(x)\sin(y)$ when $x = u + v$ and $y = u^2 - v^2$.

$$\frac{\partial z}{\partial v} =$$

(b) Assume that $F(x,y,z) = 0$ defines z implicitly as a function of x and y. Show that if $\partial F/\partial z \neq 0$, then

$$\frac{\partial z}{\partial x} = -\frac{\partial F/\partial x}{\partial F/\partial z}.$$

8. (10 pts.). (a) Use limit laws and continuity properties to evaluate the following limit.

$$\lim_{(x,y)\to(-1/4,\pi)} (xy^2\sec^2(xy)) =$$

(b) Evaluate the limit, if it exists, by converting to polar coordinates.

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{\sqrt{x^2+y^2}} =$$

9. (10 pts.) (a) Calculate $\partial z/\partial x$ using implicit differentiation when $3x^2 + 4y^2 + \tan(z) = 12$. Leave your answer in terms of x, y, and z.

(b) Find all second-order partial derivatives for the function $f(x,y) = x^3y^4$. Label correctly.

10. (10 pts.) (a) Compute the total differential dz when

$$z = \tan^{-1}(xy).$$

$dz =$

(b) Assume $f(1,-2) = 4$ and $f(x,y)$ is differentiable at $(1,-2)$ with $f_x(1,-2) = 2$ and $f_y(1,-2) = -3$. Obtain an equation for the plane tangent to the graph of f at $P(1,-2,4)$.

Silly 10 Point Bonus: (a) State the definition of differentiability for a function of two variables. [You may either state the definition found in the text or the one given by the instructor in class.] (b) Then using only the definition you stated, show the function

$$f(x,y) = x^2 + y^2$$

is differentiable at any point (x,y) in the plane. Say where your work is, for it won't fit here.