Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Generic vector objects must be denoted by using arrows. Since the answer really consists of all the magic transformations, do not "box" your final results. Show me all the magic on the page neatly.

1. (10 pts.) Evaluate the following iterated integral.

$$\int_0^{\ln(2)}\int_0^1 xye^{y^2x}\,dy\,dx =$$

2. (10 pts.) Convert the given iterated integral into an iterated integral in polar coordinates that has the same numerical value and is easier to evaluate, perhaps. Do not attempt to evaluate the iterated integrals. Assume $a > 0$.

$$\int_0^a\int_0^{\sqrt{a^2-x^2}} \frac{1}{\sqrt{1+x^2+y^2}}\,dy\,dx =$$

3. (10 pts.) Express the integral as an equivalent integral with the order of integration reversed. Sketching the region is a key piece of the puzzle.

$$\int_0^1 \int_{\sin^{-1}(y)}^{\pi/2} f(x,y)\ dx\,dy =$$

4. (10 pts.) Set up, but do not attempt to evaluate, a triple iterated integral in cartesian coordinates that would be used to find the volume of the solid G bounded below by the elliptic paraboloid $z = 4x^2 + y^2$ and above by the cylindrical surface $z = 4 - 3y^2$, [Hint: The upper and lower surfaces are on a platter with a cherry on top. Find the projection onto the xy-plane of the curve obtained when the two surfaces intersect.]

$$\iiint_G 1\ dV =$$

5. (10 pts.) Write down the triple iterated integral in cylindrical coordinates that would be used to compute the volume of the solid G whose top is the plane $z = 9$ and whose bottom is the paraboloid $z = x^2 + y^2$. Do not attempt to evaluate the integral.

$$\iiint_G 1\, dV =$$

6. (10 pts.) Locate all relative extrema and saddle points of the following function.

$$f(x,y) = x^2 + 8y^2 - x^2y$$

Use the second partials test in making your classification. (Fill in the table below after you locate all the critical points.)

Crit.Pt.	f_{xx} @ c.p.	f_{yy} @ c.p.	f_{xy} @ c.p.	D @ c.p.	Conclusion

7. (5 pts.) Write down the triple iterated integral in spherical coordinates that would be used to compute the volume of the solid G bounded above by the sphere defined by $\rho = 4$ and below by the cone defined by $\phi = \pi/3$. Do not attempt to evaluate the interated integrals.

$$\iiint_G 1 \, dV =$$

8. (15 pts.) Let $f(x,y) = x^2y^2$ on the closed unit disk defined by $x^2 + y^2 \leq 1$. Find the absolute extrema and where they occur. Use Lagrange multipliers to analyze the function on the boundary. Do not neglect the interior of the disk in doing your analysis!!

9. (10 pts.) Reveal all the details in using the substitution $u = x/a$ and $v = y/b$ with $a > 0$ and $b > 0$ to evaluate the integral below, where R is the elliptical region defined by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1.$$

[Hint: Remember that such a substitution defines T^{-1}, not T.]

$$\iint_R 1 \; dA = \iint_R 1 \; dA_{x,y}$$

$$=$$

10. (10 pts.) Compute the surface area of the portion of the paraboloid defined by $z = 16 - x^2 - y^2$ that lies above the xy-plane.

Silly 10 Point Bonus: Become a polar explorer. Compute the exact value of the double integral

$$\iint_R x \; dA$$

where R is the region in the first quadrant bounded by the lines $y = x/3$ and $y = 2x$ and the circle centered at the origin with a radius of 2. Say where your work is, for it won't fit here.