
General directions: *Read each problem carefully and do exactly what is requested. Full credit will be awarded only if you show all your work neatly, and it is correct. Use complete sentences and use notation correctly. Be very careful. Remember that what is illegible or incomprehensible is worthless. Since the answer really consists of all the magic transformations, do not box your final result. Show me all the magic on the page. Communicate.*

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1. (90 pts.) Solve each of the following differential equations or initial value problems. Show all essential work neatly and correctly. [15 points/part]

(a) $y' + 2xy = 8x \;\; ; \; y(0) = 10$

(b) $2 \cdot (y^2 - 1)dx + 20 \cdot \sec(x)dy = 0$

(c) $(y^2 + xy + x^2)dx - (xy)dy = 0$

(d) $7y' + x^{-1}y = 12x^2y^{-6}$ for $x > 0$

(e) $(x + y - 8)dx + (x - y + 4)dy = 0$

(f) $y' = f(x)$, where $f(x) = \begin{cases} 2x & , \text{for } 0 \le x < 3 \\ 6 & , \text{for } 3 \le x \end{cases}$

and $y(0) = 1$.

2. (10 pts.) (a) Obtain the differential equation and initial condition needed to solve the following word problem. State what your variables represent using complete sentences. (b) Next, solve the initial value problem. (c) Then, answer the last part of the question. [For (c), the exact value in terms of natural logs will suffice.]

//A tank initially contains 50 gallons of pure water. Starting at time t = 0, a brine containing 2 pounds of dissolved salt per gallon flows into the tank at a rate of 3 gallons per minute. Suppose the mixture is kept uniformly mixed by constant stirring and flows out of the tank at the same rate at which it enters. When will the tank contain 10 pounds of dissolved salt?//