

---

**General directions:** Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , " $\Rightarrow$ " denotes "implies" , and " $\Leftrightarrow$ " denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.

---

1. (10 pts.) The factored auxiliary equation of a certain homogeneous linear O.D.E. with real constant coefficients is as follows:

$$(m - \pi)^3(m - (2+i))^2(m - (2-i))^2 = 0$$

(a) (5 pts.) Write down the general solution to the differential equation. **[WARNING: Be very careful. This will be graded Right or Wrong!!]** (b) (5 pt.) What is the order of the differential equation?

---

2. (15 pts.) Given that  $f(x) = \sin(2x)$  is a solution of the homogeneous linear O.D.E.  $y'' + 4y = 0$ , using only the method of reduction of order, obtain a second, linearly independent solution. **[WARNING: No reduction, no credit!! Show all steps of this neatly while using notation correctly.]**

---

3. (10 pts.) Set up the correct linear combination of undetermined coefficient functions you would use to find a particular solution,  $y_p$ , for the O.D.E.

$$y'' - y' = 10x^2 - 7\sin(x) - 32xe^x.$$

[**Warning:** (a) If you skip a critical initial step, you will get no credit!! (b) Do not waste time attempting to find the numerical values of the coefficients!!]

---

4. (15 pts.) Using the method of variation of parameters, not the method of undetermined coefficients, find a particular integral,  $y_p$ , of the differential equation

$$y'' - y = 10e^x.$$

[**Hint:** Read this problem twice and do exactly what is asked to avoid heartbreak!! Do not obtain  $y_p$  using the method of undetermined coefficients. Do not waste time getting the general solution.]

---

5. (15 pts.) Write down the general solution to each of the following linear constant coefficient homogeneous equations.

(a)  $y'' - 10y' + 25y = 0$

(b)  $y'' - y' - 20y = 0$

(c)  $d^4y/dx^4 + 9(dy^2/dx^2) = 0$

---

6. (10 pts.) Very carefully obtain the general solution to the following Euler-Cauchy O.D.E.:

$$x^2y''(x) - 4xy'(x) + 6y(x) = 8 \cdot \ln(x)$$

---

7. (15 pts.) Suppose

$$y(x) = \sum_{k=0}^{\infty} c_k x^k$$

is a solution of the homogeneous second order linear equation  $y'' - 10x^2y = 0$ . (a) Obtain the recurrence formula for the coefficients of  $y(x)$ . (b) Which coefficients must be zero?? (c) If  $y(x)$  also satisfies the initial conditions  $y(0) = 0$  and  $y'(0) = 1$ , what are the values of  $c_2$ ,  $c_3$ ,  $c_4$ , and  $c_5$ ??

---

8. (10 pts.) Obtain the solution to the following initial value problem:

$$y'' - y' = 2 \sin(x)$$

$$y(0) = -1, \text{ and } y'(0) = 1$$

---

**Silly 10 Point Bonus:** Show that the following set of two functions is linearly independent and yet cannot be a fundamental set of solutions for any homogeneous second order linear O.D.E. on the whole real line:  $\{ x^3, |x|^3 \}$ . Say where your work is!