General directions: *Read each problem carefully and do exactly what is requested. Full credit will be awarded only if you show all your work neatly, and it is correct. Use complete sentences and use notation correctly. Be very careful. Remember that what is illegible or incomprehensible is worthless. Since the answer really consists of all the magic transformations, do not box your final result. Show me all the magic on the page. Communicate.*

1. (90 pts.) Solve each of the following differential equations or initial value problems. Show all essential work neatly and correctly. [15 points/part]

(a) $y' + 2x^{-1}y = 8x \; ; \; y(1) = 10$

(b) $2 \cdot \csc(y)dx + 20 \cdot (x^2 - 1)dy = 0$

(c) $(2x^2 + y^2)dx + (x^2 - xy)dy = 0$

(d) $5 \cdot y' + x^{-1}y = 8 \cdot x^2 \cdot y^{-4}$ for $x > 0$

(e) $(y \cdot \cos(xy) - 6x)dx + (x \cdot \cos(xy) + 8y)dy = 0$

(f) $y' = f(x)$, where $f(x) = \begin{cases} 6 & \text{, for } 0 \le x < 3 \\ 2x & \text{, for } 3 \le x \end{cases}$

and $y(0) = -3$.

2. (5 pts.) Very neatly provide the verification that $2x^3 + 6xy^2 = 2$ is an implicit solution of the differential equation $2xy \cdot y' + x^2 + y^2 = 0$ on the interval $0 < x < 1$.

3. (5 pts.) It is known that every solution to the differential equation $y'' - 4 \cdot y = 0$ is of the form

$$y = c_1 \cdot e^{2x} + c_2 \cdot e^{-2x}.$$

Which of these functions satisfies the initial conditions $y(0) = 2$ and $y'(0) = -4$?? [Hint: Determine c_1 and c_2 by solving an appropriate linear system. Don't waste time verifying y, above, is a solution.]

Silly 10 Point Bonus: (a) The Fundamental Theorem of Calculus provides a neat formal solution involving a definite integral with respect to the variable 't' to the following dinky IVP:

$$y'(x) = \exp(x^2) \text{ and } y(0) = 1.$$

What is that solution? (b) Unfortunately $g(x) = \exp(x^2)$ cannot be integrated in elementary terms. Use the answer to (a), the Maclaurin series for e^x, and term-wise integration, to obtain a power series solution to the IVP. [Say where your work is! You don't have room here!]