
General directions: *Read each problem carefully and do exactly what is requested. Full credit will be awarded only if you show all your work neatly, and it is correct. Use complete sentences and use notation correctly. Be very careful. Remember that what is illegible or incomprehensible is worthless. Since the answer really consists of all the magic transformations, do not box your final result. Show me all the magic on the page. Communicate.*

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1. (40 pts.) Solve each of the following second order differential equations or initial value problems. Be very careful. Show all essential work. Do not write nonsense.

(a) $y'' - 4y' - 21y = 0$

(b) $d^3y/dx^3 + 9(dy/dx) = 0$

(c) $y'' - 18y' + 81y = 0$

(d) $y'' - y' = 4e^x$; $y(0) = -2$, $y'(0) = 1$

2. (5 pts.) The factored auxiliary equation of a certain homogeneous linear O.D.E. with real constant coefficients is as follows:

$$m^3(m - \pi)(m - (1+5i))^3(m - (1-5i))^3 = 0$$

(a) (3 pts.) Write down the general solution to the differential equation. [**WARNING: Be very careful. This will be graded Right or Wrong!!**] (b) (2 pt.) What is the order of the differential equation?

3. (10 pts.) Given that $f(x) = \sin(2x)$ is a solution of the homogeneous linear O.D.E. $y'' + 4y = 0$, use only the method of reduction of order to find a second, linearly independent solution. [**WARNING: No reduction, no credit!! Show all steps of this neatly while using notation correctly.**]

4. (20 pts.) **Using the method of variation of parameters, not the method of undetermined coefficients**, find a particular integral, y_p, of the differential equation

$$y'' - 4y' = 10.$$

[Hint: Read this problem twice and do exactly what is asked to avoid heartbreak!! *Do not obtain* y_p *using the method of undetermined coefficients*.]

5. (5 pts.) The following differential equation may be solved by either performing a substitution to reduce it to a separable equation or by performing a different substitution to reduce it to a homogeneous equation. Display the substitution and perform the reduction, but **do not attempt to solve the separable or homogeneous equation you obtain**.

$$(x - y - 8)dx + (2x - 2y + 2)dy = 0$$

6. (10 pts.) Set up the correct linear combination of undetermined coefficient functions you would use to find a particular solution, y_p, for the O.D.E.

$$y'' - 3y' = 10x^2 - 7\sin(x) - 32xe^{3x}.$$

Do not attempt to actually find the numerical values of the coefficients!!

7. (10 pts.) (a) Obtain the differential equation and initial condition needed to solve the following word problem. State what your variables represent using complete sentences. (b) Next, solve the initial value problem. (c) Then, answer the last part of the question. This will probably involve a second equation relating dependent and independent variables. [For (c), the exact value in terms of natural logs will suffice.]

//Assume Newton's Law of Cooling: A body with temperature of 100 °F is placed at time t = 0 in a medium maintained at a temperature of 20 °F. If, at the end of 10 minutes the temperature of the body is 50 °F, when will the body be 30 °F??//

Silly 10 Point Bonus: Magically obtain the following antiderivative without integrating by parts or any other way!!

$$\int \sin(x) \cdot e^x dx = y$$