Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Clearly show me the all the magic on the page.* Test #:

1. (40 pts.) Without evaluating any integrals and using only the table provided, properties of the Laplace transform, and appropriate function identities, obtain the Laplace transform of each of the functions that follows:

(a) $h(t) = \begin{cases} -5 & , 0 < t < 5 \\ 25 & , 5 < t < 15 \\ 2 & , 15 < t \, . \end{cases}$

$\mathcal{L}\{h(t)\}(s) =$

(b) $g(t) = \pi \cdot t \cdot e^{2t} \cdot \cos(t)$

$\mathcal{L}\{g(t)\}(s) =$

(c) $(f*g)(t)$, when $f(t) = 2 \cdot \sin(3t)$ and $g(t) = 2 \cdot e^{-5t} \cdot t^4$

$\mathcal{L}\{(f*g)(t)\}(s) =$

1.

(d) $f(t) = \begin{cases} 16 & , 0 < t < 2 \\ 8t & , 2 < t \end{cases}$

$\mathfrak{L}\{f(t)\}(s) =$

2. (10 pts.) (a) The Laplace transform of the following 2-periodic function may be written in terms of a definite integral. Simply express the transform in terms of the appropriate definite integral, but do not attempt to evaluate that definite integral.

$f(t) = \begin{cases} t & , \text{ for } 0 \le t < 1 \\ 2 - t & , \text{ for } 1 \le t < 2, \end{cases}$

and $f(t) = f(t + 2)$ for $t \ge 0$.

$\mathfrak{L}\{f(t)\}(s) =$

(b) The following sum of definite integrals can be realized as the Laplace transform of a certain function g(t) defined for $t \ge 0$. Provide the precise definition of that function g.

$$\int_0^1 te^{-st}dt + \int_1^2 (2-t)e^{-st}dt = \mathfrak{L}\{g(t)\}(s), \text{ where}$$

$g(t) =$

Warning: *Do not attempt to evaluate the Laplace transform of g.*

3. (15 pts.) Suppose that the Laplace transform of the solution to a certain initial value problem involving a linear differential equation with constant coefficients is given by

$$\mathcal{L}\{y(t)\}(s) = \frac{se^{-\pi s}}{s^2 + 25} + \frac{10 \cdot s}{(s-2)^2 + 36} .$$

What’s the solution, $y(t)$, to the IVP??

$y(t) =$

4. (15 pts.) Using only the Laplace transform machine, very carefully solve the following very dinky first order initial value problem:

$$y' = f(t) \text{ , where } f(t) = \begin{cases} 6 & \text{, for } 0 \le t < 3 \\ 2t & \text{, for } 3 \le t \end{cases}$$

and $y(0) = -3$.

5. (10 pts.) Very neatly transform the given initial value problem into a linear system in $\mathcal{L}\{x\}$ and $\mathcal{L}\{y\}$ and stop. Do not attempt to solve for $\mathcal{L}\{x\}$ or $\mathcal{L}\{y\}$.

I.V.P.: $2x'(t) + y(t) = 12t^2$

$y'(t) - 3x(t) = 8 \cdot e^{-4t}\delta(t - 5)$, $x(0) = 1$, $y(0) = -2$

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6. (10 pts.) Consider the following initial value problem:

$y'(t) - 3y(t) = h(t)$ and $y(0) = 0$,

$$\text{where } h(t) = \begin{cases} 10\sin(t), & \text{for } 0 < t < 2\pi \\ 0, & \text{for } t > 2\pi. \end{cases}$$

Suppose that

$$y(t) = \begin{cases} e^{3t} - \cos(t) - 3\sin(t), & \text{for } 0 \le t < 2\pi \\ e^{3t}, & \text{for } t \ge 2\pi. \end{cases}$$

(a) Show that y satisfies the ODE when $0 < t < 2\pi$.

(b) Show that y satisfies the ODE when $t > 2\pi$.

(c) Show that y satisfies the initial condition.

(d) Why wouldn't you want to call y *the solution* to the IVP?

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10 Point Bonus: If $\mathcal{L}\{f(t)\}(s) = \dfrac{1/s}{1 + e^{-s}}$ *for* $s > 0$, what's f(t), assuming the Laplace beast coexists with the series shaman?