
General directions: Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , " \Rightarrow " denotes "implies" , and " \Leftrightarrow " denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.

Silly 10 Point Bonus: What linear homogeneous ODE with constant coefficients has a fundamental set of solutions given by $\{ e^x, e^x \sin(x), e^x \cos(x) \}$ Say where your work is!

If $\{ e^x, e^x \sin(x), e^x \cos(x) \}$ is a fundamental set of solutions to a constant coefficient equation with real coefficients, we can see that the auxiliary equation has as its roots $1, 1 + i$, and $1 - i$. Consequently, we can rebuild the desired polynomial by a routine multiplication:

$$p(m) = (m - 1)(m - (1 + i))(m - (1 - i)) = \dots = m^3 - 3m^2 + 4m - 2$$

Ah...The ODE, that's easy now:

$$y''' - 3y'' + 4y' - 2y = 0.$$

Kindly note that this is not the only way in which this problem may be attacked. You could, of course, build a little three by three linear system by substituting into an appropriate differential equation. Solving that linear system would provide the desired coefficients. o.o.