General directions: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇨" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

1. (15 pts.) Write down the general solution to each of the following linear constant coefficient homogeneous equations.

(a) $y''(x) + 4y'(x) + 4y(x) = 0$

(b) $y''(x) + 3y'(x) - 10y(x) = 0$

(c) $\frac{d^5y}{dx^5} - 4\frac{d^3y}{dx^3} = 0$

2. (10 pts.) The factored auxiliary equation of a certain homogeneous linear O.D.E. with real constant coefficients is as follows:

$$(m - \pi)^3(m - (1+2i))^2(m - (1-2i))^2 = 0$$

(a) (5 pts.) Write down the general solution to the differential equation. [**WARNING: Be very careful. This will be graded Right or Wrong!!**] (b) (5 pt.) What is the order of the differential equation?

3. (10 pts.) It turns out that the nonzero function $f(x) = e^x$ is a solution to the homogeneous linear O.D.E.

$$y'' - y = 0.$$

Using only the method of reduction of order, show how to obtain a second, linearly independent solution to this equation.

[**WARNING: No reduction, no credit!! Show all steps of this neatly while using notation correctly. You are being graded on the journey, not the destination.**]

4. (15 pts.) Very carefully obtain the solution to the initial value problem that follows.

$$(*)\begin{cases} x^2y''(x) + 4xy'(x) + 2y = 4\ln(x)\ ; \\ \\ \qquad y(1) = 1\ ,\ \ y'(1) = -1. \end{cases}$$

5. (15 pts.) Suppose

$$y(x) = \sum_{n=0}^{\infty} c_n x^n$$

is a solution of the homogeneous second order linear equation

$$y'' - xy' - y = 0.$$

(a) (10 pts.) Obtain the recurrence formula for the coefficients of $y(x)$. (b) (5 pts.) If $y(x)$ also satisfies the initial conditions $y(0) = 1$ and $y'(0) = 0$, what is the numerical value of c_4 ??

6. (15 pts.) **Using the method of variation of parameters, not the method of undetermined coefficients**, find a particular integral, y_p, of the differential equation

$$y'' + y = 1$$

[**Hint: Read this problem twice and do exactly what is asked to avoid heartbreak!!** *Do not obtain* y_p *using the method of undetermined coefficients. Do not waste time getting the general solution.*]

7. (10 pts.) Set up the correct linear combination of undetermined coefficient functions you would use to find a particular integral, y_p, of the O.D.E.

$$y'' + 4y' + 5y = e^{-2x} + \cos(x)e^{-2x}.$$

[**Warning:** *(a) If you skip a critical initial step, you will get no credit!! (b) Do not waste time attempting to find the numerical values of the coefficients!!*]

8. (10 pts.) (a) Obtain the differential equation and initial condition needed to solve the following word problem. State what your variables represent using complete sentences. (b) Next, solve the initial value problem. (c) Then, answer the last part of the question. [For (c), the exact value in terms of natural logs will suffice.]

//A tank initially contains 50 gallons of pure water. Starting at time t = 0, a brine containing 2 pounds of dissolved salt per gallon flows into the tank at a rate of 3 gallons per minute. Suppose the mixture is kept uniformly mixed by constant stirring and flows out of the tank at the same rate at which it enters. When will the tank contain 110 pounds of dissolved salt?//

Silly 10 Point Bonus: What linear homogeneous ODE with constant coefficients has a fundamental set of solutions given by { e^x, $e^x\sin(x)$, $e^x\cos(x)$ } Say where your work is!