
General directions: Read each problem carefully and do exactly what is requested. Full credit will be awarded only if you show all your work neatly, and it is correct. Write complete sentences, and use notation correctly. What is illegible or incomprehensible is worthless. Since the answer really consists of all the magic transformations, do not box your final result. Show me all the magic on the page. Communicate. Eschew obfuscation.

1. (75 pts.) Solve each of the following differential equations or initial value problems. If there is no initial condition, obtain the general solution. [15 points/part]

(a) $(y \sec^2(x) + \sec(x) \tan(x)) dx + (\tan(x) + 2y) dy = 0$

(b) $\frac{dx}{dt} + \frac{x}{t^2} = \frac{1}{t^2}$

(c) $\left(x \tan\left(\frac{y}{x}\right) + y \right) dx - x dy = 0$

(d) $\frac{dy}{dx} + \frac{y}{2x} = \frac{x}{y^3} , \quad y(1) = 2.$

(e) $x \sin(y) dx + (x^2 + 1) \cos(y) dy = 0$

2. (10 points) The following differential equation may be solved by either performing a substitution to reduce it to a separable equation or by performing a different substitution to reduce it to a homogeneous equation. Display the substitution to use and perform the reduction, **but do not attempt to solve the separable or homogeneous equation you obtain.**

$$(x - 2y + 1)dx - (4x - 8y - 6)dy = 0$$

3. (15 pts.) Solve the following first order initial value problem:

$$y'(x) + y(x) = f(x) \text{ and } y(0) = 0,$$

where

$$f(x) = \begin{cases} 1 & , \text{ if } 0 \leq x < 2 \\ 0 & , \text{ if } 2 \leq x. \end{cases}$$

Bonkers 10 Point Bonus: Pretend $A, B, C, D, E,$ and F are fixed garden variety real numbers. The ugly looking homogeneous equation

$$(*) \quad (Ax^2 + Bxy + Cy^2)dx + (Dx^2 + Exy + Fy^2)dy = 0$$

is actually very easy to solve when it turns out also to be exact. Determine exactly when this happens, with proof. Note well that you are NOT being asked to actually solve the differential equation!!

[Hint: You might want to ask $A, B, C, D, E,$ and F what exact role they might play in this game. Say where your work is! You don't have room here to explain things!]