General directions: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇨" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

1. (30 pts.) Obtain the general solution to each of the following linear homogeneous constant coefficient equations.

(a) $y''(x) - 7y'(x) + 10y(x) = 0$

(b) $y''(x) - 6y'(x) + 9y(x) = 0$

(c) $\dfrac{d^5y}{dx^5} + 25\dfrac{d^3y}{dx^3} = 0$

2. (10 pts.) Find the unique solution to the initial value problem

$$y'' + 4y = 6\sin(x)\ ;\ y(\pi/2) = -1\ ,\ y'(\pi/2) = 1$$

given that a fundamental set of solutions to the corresponding homogeneous equation is { cos(2x), sin(2x) } and a particular integral to the original ODE is

$$y_p(x) = 2\sin(x).$$

Hint: Save time. Use the stuff served on the platter with the cherry on top.

3. (10 pts.) It turns out that the nonzero function $f(x) = \sin(x)$ is a solution to the homogeneous linear O.D.E.

$$y'' + y = 0.$$

Using only the method of reduction of order, show how to obtain a second, linearly independent solution to this equation.

[**WARNING: No reduction, no credit!! Show all steps of this neatly while using notation correctly. You are being graded on the journey, not the destination.**]

4. (10 pts.) Very carefully obtain the general solution for $x > 0$ to the following ODE:

$$x^2y'' - 6y = \ln(x)$$

5. (10 pts.) The factored auxiliary equation of a certain homogeneous linear O.D.E. with real constant coefficients is as follows:

$$(m - 1)^2(m - (2i))^3(m - (-2i))^3 = 0$$

(a) (5 pts.) Write down the general solution to the differential equation. [**WARNING: Be very careful. This will be graded Right or Wrong!!**]

(b) (5 pt.) What is the order of the differential equation?

6. (10 pts.) **Using the method of variation of parameters, not the method of undetermined coefficients**, find a particular integral, y_p, of the differential equation

$$y'' - y = 2e^x$$

[**Hint: Read this problem twice and do exactly what is asked to avoid heartbreak!!** *Do not obtain* y_p *using the method of undetermined coefficients. Do not waste time getting the general solution.*]

7. (10 pts.) Set up the correct linear combination of undetermined coefficient functions you would use to find a particular integral, y_p, of the O.D.E.

$$y'' - 6y' + 8y = x^2 + e^{-2x} + e^{+4x}.$$

[**Warning:** *(a) If you skip a critical initial step, you will get no credit!! (b) Do not waste time attempting to find the numerical values of the coefficients!!*]

8. (10 pts.) (a) Assuming Newton's Law of Cooling is applicable, obtain the differential equation and any additional equations that the solution must satisfy to solve the following word problem. State what your variables represent using complete sentences. (b) Next, solve the initial value problem. (c) Then, answer the last part of the question. [For (c), the exact value in terms of natural logs will suffice.]

//A body with temperature of 100 °F is placed at time t = 0 in a medium maintained at a temperature of 40 °F. If, at the end of 10 minutes the temperature of the body is 90 °F, when will the body be 50 °F??//

Silly 10 Point Bonus: Show how to magically solve the following integral equation without integrating --- either by parts or any other way!!

$$y = \int x^2 e^x \, dx$$

[Say where your work is, for it won't fit here.]