Read Me First: Communicate. Show all essential work very neatly and use correct notation when presenting your computations and arguments. Write using complete sentences. Show me the all magic on the page. Eschew obfuscation.

Test #:

1. (10 pts.) Locate and classify the singular points of the following second order homogeneous O.D.E. Use complete sentences to describe the type of points and where they occur.

$$(x^2+x)^2y'' + xy' + (x+1)y = 0$$

2. (15 pts.) (a) If $f(t)$ and $g(t)$ are piece-wise continuous functions defined for $t \geq 0$, what is the definition of the convolution of f with g, $(f*g)(t)$??

$(f*g)(t) =$

(b) Suppose $f(t) = 6e^{3t}$ and $g(t) = 2e^{t}$.

Using only the definition of the convolution as a definite integral, not some fancy transform shenanigans, compute $(f*g)(t)$.

$(f*g)(t) =$

(c) Suppose $h(t) = (f*g)(t)$, *where* $f(t) = t^2$ *and* $g(t) = \sin(2t)$.

Using the table, compute the Laplace transform of h.

$\mathfrak{L}\{h(t)\}(s) =$

3. (15 pts.) Suppose

$$y(x) = \sum_{n=0}^{\infty} c_n x^n$$

is a solution of the homogeneous second order linear equation

$$y'' - y' + x^2 y = 0.$$

Very neatly obtain the recurrence formula(s) needed to determine the coefficients of $y(x)$. DO NOT SPEND TIME ATTEMPTING TO GET THE NUMERICAL VALUES OF THE COEFFICIENTS.

4. (10 pts.) (a) Suppose that $f(t)$ is defined for $t > 0$. What is the definition of the Laplace transform of f in terms of a definite integral??

$\mathfrak{L}\{f(t)\}(s) =$

(b) Using only the definition, not the table, compute the Laplace transform of

$$f(t) = \begin{cases} 0 \ , & if \ \ 0 < t < 1 \\ 2 \ , & if \ \ 1 < t. \end{cases}$$

$\mathfrak{L}\{f(t)\}(s) =$

5. (10 pts.) The equation below has a regular singular point at $x_0 = 0$.

$$x^2y'' + 3xy' + (x^2-8)y = 0$$

(a) Obtain the indicial equation for the ODE at $x_0 = 0$ and its two roots.
(b) Then use all the information available and Theorem 6.3 to say what the two non-trivial linearly independent solutions given by the theorem look like without attempting to obtain the coefficients of the power series.

6. (10 pts.) Compute $f(t) = \mathfrak{L}^{-1}\{F(s)\}(t)$ when

(a) $F(s) = \dfrac{5s+17}{s^2+4s+13}$

(b) $F(s) = \dfrac{5}{s^2+4s+4}$

7. (5 pts.) Circle the letter corresponding to the correct response: If $F(s) = \mathfrak{L}\{t\sin(bt)e^{at}\}(s)$,then $F(s)$ =

(a) $\left[\dfrac{1}{s^2}\right]\cdot\left[\dfrac{b}{s^2+b^2}\right]\cdot\left[\dfrac{1}{s-a}\right]$

(b) $\dfrac{d}{ds}\left[\dfrac{b}{(s-a)^2+b^2}\right]$

(c) $\dfrac{2b(s-a)}{((s-a)^2+b^2)^2}$

(d) $\left[\dfrac{2bs}{(s^2+b^2)^2}\right]\cdot\left[\dfrac{1}{s-a}\right]$

(e) $\left[\dfrac{b}{s^2+b^2}\right]\cdot\left[\dfrac{1}{(s-a)^2}\right]$

(f) None of (a) through (e).

8. (15 pts.) Transform the given initial value problem into an algebraic equation in $\mathfrak{L}\{y\}$ and solve for $\mathfrak{L}\{y\}$. Do not take inverse transforms and do not attempt to combine terms over a common denominator. Be very careful.

$$y''(t) - 6y'(t) + 5y(t) = 0 \; ; \quad y(0) = 3 \; , \quad y'(0) = 7 \quad .$$

9. (10 pts.) The solution to a certain linear ordinary differential equation with coefficient functions analytic at $x_0 = 0$ is of the form

$$y(x) = \sum_{n=0}^{\infty} c_n x^n$$

where the coefficients satisfy the following equations:

$$-c_2 = 0, \quad c_0 + 3c_1 - 6c_3 = 0, \text{ and}$$

$$c_n = \frac{(n-2)nc_{n-2} + c_{n-3}}{n(n-1)} \quad \text{for all } n \geq 4.$$

Determine the exact numerical value of the coefficients c_0, c_1, c_2, c_3, and c_4 for the

particular solution that satisfies the initial conditions $y(0) = 1$ and $y'(0) = 0$.

c_0 =

c_1 =

c_2 =

c_3 =

c_4 =

Silly 10 Point Bonus: Would you believe that you have already done the hard work in evaluating the following improper integral??

$$I = \int_0^{\infty} \left[\int_0^{t} x^2 \sin(2(t-x))\, e^{-2t}\, dx \right] dt$$

What is the numerical value of I? Say where your work is, for it might not fit here.